HEAT CAPACITY, TIME CONSTANT, AND SENSITIVITY OF EARTH'S CLIMATE SYSTEM

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June 2007
Minor typographical errors corrected, August 19, 2007

Accepted for publication in
Journal of Geophysical Research

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Heat capacity, time constant, and sensitivity of Earth's climate system

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[Revised 2007-06-14]

ABSTRACT. The equilibrium sensitivity of Earth's climate is determined as the quotient of the relaxation time constant of the system and the pertinent global heat capacity. The heat capacity of the global ocean, obtained from regression of ocean heat content vs. global mean surface temperature, GMST, is 14 ± 6 W yr m\(^{-2}\) K\(^{-1}\), equivalent to 110 m of ocean water; other sinks raise the effective planetary heat capacity to 17 ± 7 W yr m\(^{-2}\) K\(^{-1}\) (all uncertainties are 1-sigma estimates). The time constant pertinent to changes in GMST is determined from autocorrelation of that quantity over 1880-2004 to be 5 ± 1 yr. The resultant equilibrium climate sensitivity, 0.30 ± 0.14 K/(W m\(^{-2}\)), corresponds to an equilibrium temperature increase for doubled CO\(_2\) of 1.1 ± 0.5 K. The short time constant implies that GMST is in near equilibrium with applied forcings and hence that net climate forcing over the twentieth century can be obtained from the observed temperature increase over this period, 0.57 ± 0.08 K, as 1.9 ± 0.9 W m\(^{-2}\). For this forcing considered the sum of radiative forcing by incremental greenhouse gases, 2.2 ± 0.3 W m\(^{-2}\), and other forcings, other forcing agents, mainly incremental tropospheric aerosols, are inferred to have exerted only a slight forcing over the twentieth century of -0.3 ± 1.0 W m\(^{-2}\).

Index terms:

1610 GLOBAL CHANGE Atmosphere
1635 GLOBAL CHANGE Oceans
3305 ATMOSPHERIC PROCESSES Climate change and variability
4930 Greenhouse gases
4902 Anthropogenic effects
1. Introduction

Changes in Earth's radiation budget due to human influences are of major current concern [IPCC, 2007]. Of principal concern is the change in climate due to increased concentrations of carbon dioxide because of the long lifetime of excess CO₂ in the atmosphere-ocean system and the intrinsic connection of excess CO₂ to energy production through fossil fuel use. While there are many indicia of climate change that may result from increased atmospheric concentrations of CO₂, the principal index of change is the increase in global mean temperature, especially as this change is the driver of, or is closely correlated with, changes in other key components of the climate system such as atmospheric water vapor content, the nature and extent of clouds, land and sea ice cover, and sea level.

Although climate change has been the subject of intense research for the past three decades, little progress has been made in decreasing the uncertainty associated with equilibrium sensitivity, the equilibrium change in global mean surface temperature GMST that would result from a sustained radiative forcing, typically expressed as that which would result from a doubling of atmospheric CO₂, Figure 1. While the apparent slow rate of progress in decreasing this uncertainty does not reflect the many advances in understanding of the many processes that need to be represented in global climate models, it nonetheless suggests the utility if not the necessity of alternative approaches to determining climate sensitivity on a time scale such that this determination can be made on a way that it can usefully inform policymaking. For a recent review of approaches to determine climate sensitivity and examination of constraints on the magnitude of this sensitivity see Annan and Hargreaves [2006]. Here an initial attempt is made to determine climate sensitivity through energy balance considerations that are based on the time dependence of GMST and ocean heat content over the period for which instrumental measurements are available.

This paper consists of an exposition of the single-compartment energy balance model that is used for the present empirical analysis, empirical determination of the effective planetary heat capacity that is coupled to climate change on the decadal time scale from trends of GMST and ocean heat content, empirical determination of the climate system time constant from analysis of autocorrelation of the GMST time series, and the use of these quantities to provide an empirical estimate of climate sensitivity. These results are then used to draw inferences about climate forcing over the twentieth century, for which reliable estimates of change in global mean temperature are available.
2. Earth's energy budget and its response to perturbations

Earth's climate system consists of a very close radiative balance between absorbed shortwave (solar) radiation $Q$ and longwave (thermal infrared) radiation emitted at the top of the atmosphere $E$.

\[ Q \approx E \quad (1) \]

The global and annual mean absorbed shortwave irradiance $Q = \gamma J$, where $\gamma$ is the mean planetary coalbedo (complement of albedo) and $J$ is the mean solar irradiance at the top of the atmosphere (1/4 the Solar constant) $\approx 343 \text{ W m}^{-2}$. Satellite measurements yield $Q \approx 237 \text{ W m}^{-2}$ [Ramanathan 1987; Kiehl and Trenberth, 1997], corresponding to $\gamma \approx 0.69$. The global and annual mean emitted longwave irradiance may be related to the global and annual mean surface temperature GMST $T_s$ as $E = \varepsilon \sigma T_s^4$ where $\varepsilon$ is the effective planetary longwave emissivity, defined as the ratio of global mean longwave flux emitted at the top of the atmosphere to that calculated by the Stefan-Boltzmann equation at the global mean surface temperature; $\sigma$ is the Stefan-Boltzmann constant.

Within this single-compartment energy balance model [e.g., North et al., 1981; Dickinson, 1982; Hansen et al., 1985; Harvey, 2000; Andreae et al., 2005, Boer et al., 2007] an energy imbalance $Q - E$ arising from a secular perturbation in $Q$ or $E$ results in a rate of change of the global heat content given by

\[ \frac{dH}{dt} = Q - E \quad (2) \]

where $dH/dt$ is the change in heat content of the climate system. The Ansatz of the energy balance model is that $dH/dt$ may be related to the change in GMST as

\[ \frac{dH}{dt} = C \frac{dT_s}{dt} \quad (3) \]

where $C$ is the pertinent heat capacity. Here it must be stressed that $C$ is an effective heat capacity that reflects only that portion of the global heat capacity that is coupled to the perturbation on the time scale of the perturbation. In the present context of global climate change induced by changes in atmospheric composition on the decade to century time scale the pertinent heat capacity is that which is subject to change in heat content on such time scales. Measurements of ocean heat content over the past 50 years indicate that this heat capacity is dominated by the heat capacity of the upper layers of the world ocean [Levitus et al., 2005]. From (2) and (3)

\[ C \frac{dT_s}{dt} = \gamma J - \varepsilon \sigma T_s^4. \quad (4) \]
Energy balance of the climate system in response to a perturbation in radiation is restored as the climate system relaxes to a new steady state. As $T_s$ increases in response to an imposed positive change in the radiation budget (positive forcing), the outgoing longwave flux increases, limiting the resulting increase in temperature rise. Conventionally for small perturbations a linear relation is assumed between steady-state change in $T_s$, $\Delta T_s(\infty)$, and an imposed forcing $F = \Delta(Q - E)$:

$$\Delta T_s(\infty) = \lambda_s^{-1} F,$$

where $\lambda_s^{-1}$ is denoted the equilibrium climate sensitivity.

The time-dependent response of Earth's average surface temperature to an imposed radiative forcing is generally characterized in terms of the $e$-folding time $\tau$ that would characterize relaxation to a new steady state following a perturbation. For a small step-function radiative forcing $F$ imposed at time $t = 0$, solution of Eq (4) [Dickinson, 1982; Hansen et al., 1985; Harvey, 2000] yields

$$\Delta T_s(t) = \lambda_s^{-1} F(1 - e^{-t/\tau}),$$

where the time constant is related to the equilibrium sensitivity by the system heat capacity as

$$\tau = C \lambda_s^{-1}.$$

Knowledge of Earth's equilibrium climate sensitivity $\lambda_s^{-1}$ and time constant $\tau$ is essential to interpreting climate change over the anthropocene era and to predicting future climate change in response to assumed forcings. These quantities are not well known. The present estimate of Earth's equilibrium climate sensitivity, expressed as the increase in global mean surface temperature for doubled CO$_2$, is $\Delta T_{2x} \approx 3.1^\pm 1.5$ K [IPCC, 2007], corresponding, for the forcing of doubled CO$_2$, $F_{2x} = 3.7$ W m$^{-2}$, to $\lambda_s^{-1} = 0.8^{+0.4}_{-0.3}$ K/(W m$^{-2}$), Figure 1.

In the absence of feedbacks, i.e., $\varepsilon$ and $\gamma$ in Eq (4) held constant, solution of Eq (4) for a small step-function forcing yields

$$\lambda_0^{-1} = T_s / 4 J \gamma \quad \text{and} \quad \tau_0 = C T_s / 4 J \gamma.$$

For the present global mean surface temperature $T_s = 288$ K, $\lambda_0^{-1} = 0.30$ K/(W m$^{-2}$). A planetary climate sensitivity that is greater than that calculated for constant $\gamma$ and $\varepsilon$ would be indicative of positive feedback. In the energy balance framework a positive feedback would result from a decrease in effective emissivity of the planet $\varepsilon$ with increasing GMST because of increased water-vapor mixing ratio in the atmosphere, and/or
an increase in planetary coalbedo $\gamma$ due to decrease in cloudiness with increasing GMST. Such feedbacks result in $\lambda^{-1}$ and $\tau$ being increased above their values by a feedback factor $f$ such that

$$\lambda_s^{-1} = f\lambda_0^{-1} \quad \text{and} \quad \tau = f\tau_0.$$  \hspace{1cm} (9)

where

$$f = \left(1 - \frac{1}{4} \frac{d\ln\gamma}{d\ln T_s} + \frac{1}{4} \frac{d\ln\varepsilon}{d\ln T_s}\right)^{-1}.$$  \hspace{1cm} (10)

The value of the feedback factor, and hence of the equilibrium sensitivity and time constant of Earth's climate system, are not known. It is generally agreed that the feedback factor is no greater than several fold [Hansen et al., 1985], but the plausible range in $f$ admits to substantial uncertainty in $\lambda_s^{-1}$. A climate sensitivity $\lambda_s^{-1} = 0.8^{+0.4}_{-0.3}$ K/(W m$^{-2}$) would correspond to a feedback factor of $2.7^{+1.3}_{-0.9}$.

The linear energy balance model readily admits solution to a forcing that is time dependent. For a forcing that increases linearly with time $F = \beta t$,

$$\Delta T_s(t) = \beta\lambda_s^{-1}[(t - \tau) + \tau e^{-t/\tau}] ;$$  \hspace{1cm} (11)

at time following the onset of the perturbation sufficiently great that transients have died away, $t \geq 3\tau$,

$$\Delta T_s(t) \approx \beta\lambda_s^{-1}(t - \tau).$$  \hspace{1cm} (12)

If $\tau$ is short compared to the duration of the forcing, then GMST would be expected to closely follow the forcing, with little lag; and if the change in forcing were abruptly stopped the additional change in GMST would be

$$\Delta T_{\text{lag}} = \beta\lambda_s^{-1}\tau.$$  \hspace{1cm} (13)

Alternatively, if $\tau$ is long, then the change in GMST would considerably lag the forcing and temperature would continue to change substantially before the climate system reached a new steady state. In particular for a situation in which climate is being forced by increasing concentrations of greenhouse gases, the increase in temperature beyond that realized at a given observation time that might be expected in response to forcing that has been applied until that time has been denoted "unrealized" or "committed" warming that is "in the pipeline" and is attributed to "thermal inertia" [Meehl et al., 2005; Wigley, 2005; Friedlingstein and Solomon, 2005; Hansen et al., 2005]. Knowledge of which situation characterizes Earth's climate is key to interpreting climate change that has occurred over the anthropocene. If the climate system rapidly equilibrates, then climate sensitivity can be inferred from the forcing and the increase in temperature over a
given time period. In contrast, if the climate response time is long, inferring climate sensitivity in this way would lead to an estimate of sensitivity that would be too low, perhaps substantially so.

Although the energy balance model provides a framework for interpretation of the equilibrium sensitivity of Earth's climate system, the key quantities, $\lambda_s^{-1}$, $\tau$, and $C$ are not known \textit{a priori} but must be determined. Within this model the three quantities are related by Eq (7) so that determination of any two of these quantities leads to knowledge of the third. Here observations of global mean surface temperature over 1880–2004 and ocean heat content over 1956–2003 permit empirical determination of the climate system time constant $\tau$, and effective heat capacity $C$. The equilibrium sensitivity $\lambda_s^{-1}$ of Earth's climate system is determined as

$$\lambda_s^{-1} = \tau / C.$$  \hspace{1cm} (14)

3. Determination of the effective heat capacity of Earth's climate system

A useful result from the energy balance model is a relation between the rate of change of the total global heat content $H$ and the rate of change of global mean surface temperature $T_s$, Eq (3), which allows the effective heat capacity of the system to be estimated empirically as

$$C = \frac{dH / dt}{dT_s / dt}$$  \hspace{1cm} (15)

provided rate of change of both $H$ and $T_s$ are known. For this evaluation I make use of compilations of the heat content of the world ocean, as tabulated for the years 1956-2002 by Levitus \textit{et al.} [2005] for ocean depth from the surface to 300, 700, and 3000 m (denoted here L300, L700, and L3000, respectively). As noted above, this ocean heat content anomaly accounts for most but not all of increase in the heat content of the planet over this time period; hence the resulting estimate of planetary heat capacity must be increased to take into account other heat reservoirs. For GMST I use the values tabulated by the Goddard Institute of Space Studies [GISS, NASA, USA; Hansen \textit{et al.}, 1996; updated at http://data.giss.nasa.gov/gistemp/] and the Climatic Research Unit [CRU, University of East Anglia, UK; Jones and Moberg, 2003; updated at http://cdiac.esd.ornl.gov/trends/temp/jonescru/jones.html]. Time series of these quantities are shown Figure 2. The rates of change of temperature anomaly and heat content anomaly with time were evaluated as the slopes of linear fits to each of the data sets (Table 1). Uncertainties associated with the slopes were evaluated taking into account temporal autocorrelation by multiplying the conventional variance in slope by the factor \([1 + 2\sum r(\Delta t)]\), where $r(\Delta t)$ is the autocorrelation coefficient of the time series as a function of lag time $\Delta t$ and the sum is taken up to the last nonnegative value of $r$ [Leith, 1973]. The slopes of the ocean
heat content data \( dH/dt \) based on temperature sounding data represent an unambiguous measure of ocean heating rate to the indicated depths. Despite the order-of-magnitude greater heat capacity, the heating rate from 300 m to 3000 m was only about 50% greater than that from the surface to 300 m, indicative that the relevant heat capacity of the climate system is within about a factor of 2.5 of that of the first 300 meters and is thus likely to represent the great majority of the ocean that is coupled to the climate system on the multidecadal time scale examined here; for uniform penetration the heat capacity would scale roughly as the depth.

Also shown in Table 1 are values of the effective global ocean heat capacity determined by Eq (15) as the ratio of the rates of increase of GMST and oceanic heat content. For each ocean depth, surface to 300, 700, and 3000 m the resulting values of effective heat capacity given for the two temperature data sets are fairly similar, reflective of the fairly close agreement of the two temperature trend compilations. The effective heat capacity increases with ocean depth indicative of the incremental amount of ocean water that is being heated. However this increase in heat capacity with depth is well less than the increase in depth itself, indicative that less of the deeper water is coupled to the surface, as expected. The relation between \( C \), \( dH/dt \), and \( dT_s/dt \) is depicted graphically in Figure 3, in which the diagonals represent constant values of \( C \); for the slightly greater value of \( dT_s/dt \) for the GISS data set than for the CRU data set the effective ocean heat capacity is slightly lower.

A potential concern with evaluating global ocean heat capacity as \( (dH/dt)/(dT_s/dt) \) that is manifested in Figure 2 arises from the relatively large fluctuation in ocean heat content compared to that in the temperature anomaly data series. The upward fluctuations in ocean heat content represent an increase in planetary heat content that is greater than the average over the time period, and correspondingly the downward fluctuations represent a loss in planetary heat content. What can give rise to such fluctuations that are evidently unforced by the surface temperature? Clearly the planetary heat balance must be fluctuating on account of changes in planetary albedo and/or effective planetary emissivity on these time scales, as these are the only means by which the heat content of the planet can change. As these changes are not forced by the surface temperature they must therefore be manifestations of internal variability of the climate system. For this reason it seems essential, for the purpose of inferring planetary heat capacity, that as long a time base as possible be used in evaluating the two slopes.

An alternative approach to evaluation of \( C \) (Method 2) is as the slope of a graph of global heat content versus GMST

\[
C = dH / dT_s.
\]
as shown in Figure 4. Here the slopes are evaluated according to the "ordinary least squares bisector method," which treats both variables (H and T) symmetrically in the least squares analysis [Isobe et al., 1990] and which was shown by those investigators to be likely to yield an unbiased estimate of the regression slope, as is desired here, rather than a biased estimate that would result from least squares regression on only one of the variables; autocorrelation was accounted for in the estimated uncertainties as above. The results of this analysis, summarized in Table 2, are quite similar to those obtained as the ratio of the slopes of H and T versus time, Table 1. However the value of C determined for ocean heat content from the surface to 3000 m is about 25% less than that obtained by Eq (15) as the ratio of the slopes of the two quantities versus time.

The overall average heat capacities given in Table 2 are the averages of the values obtained by the two methods; the uncertainties encompass the bulk of the values obtained by both methods. It must be stressed that the effective heat capacity of the global ocean determined in this way is not an intrinsic property of the climate system but is reflective of the rate of penetration of heat energy into the ocean in response to the particular pattern of forcing that Earth has experienced prior to and during the period of the measurements.

Also given in Table 2 for the several depths are the values of ocean depth that would exhibit the same heat capacity as is effectively coupled to the climate system. This coupled heat capacity is much less than the actual heat capacity of the ocean to the indicated depths, 23%, 16%, and 5%, for depths 300 m, 700 m, and 3000 m, respectively. It must be stressed that the heat penetration is not uniform globally, but is manifested in plumes in the regions of deep water formation. This is illustrated by the contours of heating in Figure 2 of Levitus et al. [2005], which also shows the much lower heating rates at depth compared to those near the surface. The relatively small increase in ocean heat content between 700 and 3000 m depth suggests that the great majority of ocean heat content is encompassed in depths less that 3000 m; the average depth of the world ocean is about 3800 m.

The present analysis indicates that the effective heat capacity of the world ocean pertinent to climate change on this multidecadal scale may be taken as 14 ± 6 W yr m⁻² K⁻¹. The effective heat capacity determined in this way is equivalent to the heat capacity of 106 m of ocean water or, for ocean fractional area 0.71, the top 150 m of the world ocean. This effective heat capacity is thus comparable to the heat capacity of the ocean mixed layer.

Estimates of the effective ocean heat capacity have previously been presented by Andreae et al. (2005) and by Frame et al. (2005), in neither instance with description of how the quantity was calculated or any statistical analysis. Evidently in both instances the effective heat capacity was inferred from the observed
change in global mean ocean heat content to the 3000 meter depth over the total time period 1958-1995 of
the Levitus et al., [2005] data, divided by the change in global mean surface temperature over the same
period. The estimate presented by Andreae et al. (2005), 1.1 ± 0.5 GJ m² K⁻¹ equivalent to 35 ± 16 W yr
m⁻² K⁻¹, is much greater than and wholly inconsistent with the value obtained here. The value presented by
Frame et al. [2005], (0.1 - 2.05) GJ m⁻² K⁻¹ (5-95% confidence) equivalent to (3.2 - 65) W yr m⁻² K⁻¹,
encompasses the value obtained here but extends both to much higher and much lower values. A possible
explanation for the discrepancies is that an approach that is based on the difference over the full time period
gives undue weight to the end members of the time series rather than relying on the totality of the data.

Following the estimate of Levitus et al. [2005] that the heat uptake of the world ocean constitutes 84% of
the total heat uptake by the climate system (other major components are heating of continental land masses,
5%; melting of continental glaciers, 5%, and heating of the atmosphere, 4%), I evaluate the global heat
capacity pertinent to climate change on the multidecadal scale as 16.7 ± 7.0 W yr m⁻² K⁻¹.

In a simulation with two coupled ocean-atmosphere climate models of the response of Earth's climate
system to the shortwave aerosol cooling forcing following the 1991 eruption of Mount Pinatubo Boer et al.
[2007] inferred effective heat capacity of the climate system to be 0.25 GJ m⁻² (8 W yr m⁻² K⁻¹), which
they noted to be comparable to the heat capacity of a mixed layer ocean of depth 50 m. A concern raised by
those investigators over the pertinence of a heat capacity determined in this way to the multidecadal time
scales associated with greenhouse forcing is the short time scale of the volcanic aerosol forcing, which they
characterized by a time constant of 8 months, resulting in relatively little penetration of the thermal signal
into the deep ocean.

The heat capacity determined from this analysis leads to a value of the time constant of the global climate
system in the absence of feedbacks (Eq 8) \( \tau_0 = 5 ± 2 \) yr.

4. Time constant of Earth's climate system from time series analysis

The second quantity necessary for empirical determination of Earth's climate sensitivity is the time constant
describing relaxation of global mean surface temperature following a perturbation. Again the intent is to
determine this quantity empirically. Here I use the framework of time series analysis to infer this time
constant from the temporal autocorrelation of GMST. This analysis rests fundamentally on the fluctuation
dissipation theorem of nonequilibrium thermodynamics [Einstein, 1905], which relates the impulse
response of a dynamic system to the fluctuations of the system. While this applicability remains an open
question, several studies, notably Leith [1975, 1978] have made the case for its applicability to Earth's
climate system. The analytical framework of the fluctuation dissipation theorem has been applied to

Under the assumption that the system behaves as a first-order Markov or autoregressive process [Leith, 1973; Allen and Smith, 1996; H. von Storch and Zwiers, 1999], for which a quantity is assumed to decay to its mean value with time constant $\tau$ but is subject also to random perturbations, the autocorrelation coefficient of the data, that is, the Pearson product-moment correlation coefficient of the time series with a copy of itself lagged by time $\Delta t$, is related to the time constant characterizing the relaxation of departures from the mean state as

$$r(\Delta t) = \exp(-\Delta t / \tau).$$  \hspace{1cm} (17)

Accordingly the relaxation time constant for any lag time $\Delta t$ up to $\Delta t$ for which $r < 0$ can be evaluated as

$$\tau(\Delta t) = -\Delta t / \ln r(\Delta t).$$  \hspace{1cm} (18)

The autocorrelation properties of times series of meteorological data have been examined by several investigators in the context of inferring the time period characterizing loss of memory of prior states, for the purpose of aducing criteria of forecast skill or determining the number of degrees of freedom pertinent to calculation of the variance of a data set under examination [Leith, 1973; Trenberth, 1985; von Storch and Zwiers, 1999; Cohn and Lins, 2005]. Autocorrelation results in the effective number of independent samples being less, often substantially so, than the number of measurements. With respect to GMST anomaly, attention has likewise previously been focused mainly on detection of trends and searches for periodic oscillations [e.g., Ghil and Vautard, 1991; Allen and Smith, 1996; Tol and Vellinga, 1998; Rybski *et al.*, 2006], with little attention having been paid to the information content of the autocorrelation pertinent to the relaxation time constant and sensitivity of the climate system. The autocorrelation of $T_s$ and the cross correlation between NH and SH hemispheric annual mean temperature was examined by Wigley *et al.* [1998] in the context of comparisons the observed time series and those obtained with general circulation models, but the implications of this autocorrelation on the time constant of the climate system were not pursued.

Here the time constant representative of the relaxation of GMST to perturbations is obtained from analysis of the autocorrelation of annual GMST anomaly $T_s(i)$, using the GISS and CRU annual global mean temperature anomaly data sets for the time period 1880–2004. The steps of the procedure are illustrated in Figure 5. First the time series was detrended by subtracting a linear fit and normalized. Detrending is necessary for nonstationary data set for which the long term trend induces an artificially long
autocorrelation that is not reflective of the inherent fluctuations [H. von Storch and Zwiers, 1999]; the consequences and implications of this detrending are examined below. For the detrended time series an autocorrelogram consisting lagged autocorrelation coefficients $r(\Delta t)$ for all $\Delta t$ was calculated. Satisfaction of the assumption of a first-order Markov process was assessed by examination of the residuals of the lag-1 regression, which were found to exhibit no further significant autocorrelation. Values of the relaxation time constant $\tau(\Delta t)$ were then calculated according to (18) for all $\Delta t$ until the first non-positive value of $r$ was encountered. As seen in Figure 5g, values of $\tau$ were found to increase with increasing lag time from about 2 years at lag time $\Delta t = 1$ yr, reaching an asymptotic value of about 5 years by about lag time $\Delta t = 8$ yr. As similar results were obtained with various subsets of the data (first and second halves of the time series; data for Northern and Southern Hemispheres, Figure 6) and for the deseasonalized monthly data, Figure 7, this estimate of the time constant would appear to be robust. The increase in $\tau$ with increasing $\Delta t$ would seem to be indicative of increased coupling to elements of the climate system having greater time constant; the leveling off of $\tau$ to a constant value of about 5 years at lag times as great as 15-18 years suggests that the time constant obtained in this way is reflective of the time constant of the climate system on a multi-decadal scale pertinent to changes over the industrial period. From the estimated uncertainties in $\tau$ at a given value of $\Delta t$, which are obtained from the estimated variance of individual values of $r$ calculated using an expression presented by Bartlett [1946], I estimate the asymptotic value of $\tau$ as $5 \pm 1$ yr.

A time constant for planetary response to climate change of just a few years would seem to run counter to the notion that climate change is a slow process, playing out over centuries if not millennia. Such a long climate system response time is noted in several studies with coupled ocean-atmosphere general circulation models (GCMs). Wetherald et al. [2001] found the increase of global mean surface temperature with a fully coupled atmosphere-ocean general circulation model to lag that in a model with a mixed-layer ocean by 20–30 years, which they took as a measure of the time constant of climate system response. Meehl et al. [2005] reported experiments with two different coupled ocean-atmosphere models in which forcing was held constant at the year-2000 level. In both experiments $T_s$ asymptoted to a final value about 0.2 K greater than the year-2000 value, with an $e$-folding time of about 30 years. Hansen et al. [2005] argue, based on the temperature sensitivity of their model and the unrealized forcing due to uptake of heat into the oceans, which they took as 0.60 W m$^{-2}$ over the past decade, that for forcing held constant at present levels, $T_s$ would increase another 0.6 K as the climate approached a new steady state. Using a simple model that is calibrated against a range of coupled atmosphere-ocean GCMs Wigley [2005] finds substantial increase in global mean surface temperature in response to a present atmospheric composition held constant continuing on time scales of 50 to 400 years.
In contrast to these studies there is a growing body of observational evidence to suggest that the time over which changes in climate can take place can be quite short, just a few years. High-resolution studies of temperature change in ice cores as inferred from isotope ratios and other variables demonstrate substantial widespread temperature change in periods as short as five to ten years [Taylor et al., 1997; NRC, 2002; Alley et al., 2003]. The view of a short time constant for climate change gains support also from records of widespread change in surface temperature following major volcanic eruptions. Such eruptions abruptly enhance planetary reflectance as a consequence of injection of light-scattering aerosol particles into the stratosphere. A cooling of global proportions in 1816 and 1817 followed the April, 1815, eruption of Mount Tambora in Indonesia. Snow fell in Maine, New Hampshire, Vermont and portions of Massachusetts and New York in June, 1816, and hard frosts were reported in July and August, and crop failures were widespread in North America and Europe – the so-called "year without a summer" (Stommel and Stommel, 1983). More importantly from the perspective of inferring the time constant of the system, recovery ensued in just a few years. From an analysis of the rate of recovery of global mean temperature to baseline conditions between a series of closely spaced volcanic eruptions between 1880 and 1920 Lindzen and Giannitsis [1998] argued that the time constant characterizing this recovery must be short; the range of time constants consistent with the observations was 2 to 7 years, with values at the lower end of the range being more compatible with the observations. A time constant of about 2.6 years is inferred from the transient climate sensitivity and system heat capacity determined by Boer et al. [2007] in coupled climate model simulations of GMST following the Mount Pinatubo eruption. Comparable estimates of the time constant have been inferred in similar analyses by others [e.g., Santer et al., 2001; Wigley et al., 2005]. There are thus numerous lines of evidence that the time scale characterizing global climate change might be as short as just a few years, as obtained by the present analysis. A concern noted by several investigators with inferences of system time constant from GMST following volcanic eruptions is that as the duration of the forcing is short, the response time of the system may not be reflective of that which would characterize a sustained forcing such as that from increased greenhouse gases because of lack of penetration of the thermal signal into the deep ocean.

The relaxation time constant of Earth's climate system determined from this analysis, $\tau = 5 \pm 1$ yr, is essentially the same as that given by the energy balance model in the absence of feedbacks (Sec. 3), $\tau_0 = 5 \pm 2$ yr. This result would seem to be indicative of little net feedback inherent in Earth's climate system.

A potential concern with the present approach to infer the time constant of the climate system arises from the detrending that was applied to the time series for GMST to remove the long-term autocorrelation arising from the long-term increase in GMST over the period of the measurements. Has the detrending, by imposing a high-pass filter on the data, resulted in a value of $\tau$ that is artificially short? To examine this I
carried out the same analysis on the non-detrended data as on the detrended data. As expected, this analysis resulted in estimates of the relaxation time constant that were substantially greater than the estimate obtained with the detrended data. However these estimates differed substantially for different subsets of the data: 15-17 yr for each of the data sets as a whole, 6 to 7 yr for the first half of the time series (1880-1942), and 8-10 yr for the second half of the data set (1943-2004). This dependence of $\tau$ on the time period examined suggests that the time constant obtained with the non-detrended data is not an intrinsic property of the climate system but does indeed reflect the long term autocorrelation in the data that results from the increase in GMST over the time period for which the data exist. For this reason I proceed in the further analysis using only the time constant obtained with the detrended data set.

5. Equilibrium sensitivity of Earth's climate system

The independent determinations of the effective heat capacity ($16.7 \pm 7$ W yr m$^{-2}$ K$^{-1}$, Sec. 3) and time constant ($5 \pm 1$ yr, Sec. 4) of Earth's climate system allow evaluation of the equilibrium sensitivity $\lambda_s^{-1}$ of Earth's climate system pertinent to climate change on the multidecadal time scale by Eq (14). The resulting climate sensitivity is $0.30 \pm 0.09$ K/(W m$^{-2}$); for forcing corresponding to doubled CO$_2$ taken as $F_{2\times} = 3.7$ W m$^{-2}$, the corresponding equilibrium increase in global mean surface temperature for doubled CO$_2$, is $\Delta T_{2\times} = 1.1 \pm 0.3$ K. (These and other results are summarized in Table 3). This climate sensitivity is much lower than current estimates, e.g., the Fourth Assessment Report of the Intergovernmental Panel on Climate Change [IPCC, 2007], $\Delta T_{2\times} = 3^{+1.5}_{-1}$ K.

Although the time constant empirically determined here is much shorter than that characterizing climate response in studies with coupled ocean-atmosphere GCMs noted above [Wetherald et al., 2001; Meehl et al., 2005; Hansen et al., 2005; Wigley, 2005], the values of time constant and the climate sensitivity are in fairly close agreement with values from a zero-dimensional energy transfer model coupled to an upwelling diffusion ocean model [Dickinson and Schaudt, 1998], which yielded an impulse response time of 5 yr and equilibrium response to CO$_2$ doubling $\Delta T_{2\times} = 2$ K.

6. Climate forcing and temperature change over the twentieth century

The rather short time constant of the climate system determined by this analysis implies that the climate system is in near equilibrium with the applied forcing. Hence the total forcing of the climate system over a given time period $F$ can be determined empirically from knowledge of the change in GMST over that period $\Delta T_s$ as $F = \Delta T_s / \lambda_s^{-1}$ with little error resulting from lag of GMST response to the forcing. Here I determine the total forcing over the twentieth century $F_{20}$ for which period the change in GMST $\Delta T_{s,20}$ is
given by Folland et al. [2001] as $0.57 \pm 0.085$ K (1-sigma); this time period is chosen to be sufficiently long that the uncertainty in $\Delta T_s$ (±15%) contributes relatively little to the uncertainty in inferred total forcing over the time period, although it is recognized that change in forcing and GMST has not been constant over this period. The total forcing of the climate system over this period inferred in this way is $F_{20} = 1.9 \pm 0.9$ W m$^{-2}$; this value may be compared to the total greenhouse gas forcing over the twentieth century, $F_{G,20} = 2.2 \pm 0.3$ W m$^{-2}$ [IPCC, 2001, Figure 6.8]. As a consistency check the average forcing of $0.019$ W m$^{-2}$ yr$^{-1}$ over the twentieth century together with the 5-year lag in temperature relative to forcing (Eq. 13) yields a lag in temperature of 0.03 K. It would thus appear that there is very little unrealized warming as a consequence of "thermal inertia" of the climate system, so for all practical purposes the climate system can be considered in steady-state (or "equilibrium") with the applied forcing.

The difference $\Delta F_{20}$ between the total forcing inferred empirically from climate sensitivity and the calculated forcing due to changes in greenhouse gases must be ascribed to forcing by other influences on long- or shortwave radiation over the twentieth century; i.e., $\Delta F_{20} = F_{20} - F_{G,20} = -0.30 \pm 0.97$ W m$^{-2}$. Consideration of secular changes in forcing over the twentieth century is aided by the fact that forcing by volcanic aerosols, as estimated by Ammann et al. [2003], was quite small ($< 0.02$ W m$^{-2}$) at both ends of the century. Although the central value of the additional forcing is small compared to greenhouse gas forcing over the twentieth century, the uncertainty range extends to include either substantial cooling (-1.3 W m$^{-2}$) or substantial warming (+ 0.7 W m$^{-2}$). The most likely candidate for much of this additional forcing would be shortwave direct and indirect forcing by tropospheric aerosols [Gregory et al., 2002; Andreae et al., 2005] although forcing by all other changes in the climate system must be taken into account [Rodhe et al., 2000], of which the most likely class would appear to be changes in surface albedo [Feddema et al., 2005]. The upper limit (one-sigma) of the (negative) forcing by tropospheric aerosols obtained in this way, -1.3 W m$^{-2}$, is consistent with other estimates of this forcing by "inverse calculations" [Anderson et al., 2003] and is broadly consistent with the range of estimates of this forcing by IPCC [2007] within the quite high uncertainty of those estimates, but is substantially less than some estimates of this forcing by "forward calculations" obtained with aerosol loadings calculated with chemical transport models [Anderson et al., 2003].

The rapid equilibration of Earth's climate system to applied forcings implies that forcings by individual forcing agents over a specified time period contribute to changes in GMST over that time period and that these temperature changes may be considered additive to yield the total change in GMST, just as the individual forcings are considered additive to yield the total forcing. Thus the greenhouse gas forcing over the twentieth century of $F_{G,20}$ results in an increase in GMST of $0.66 \pm 0.32$ K. Within the uncertainty range associated with the forcings the increase in GMST due to greenhouse gases may be substantially
offset by a countervailing decrease in GMST of as much as 0.38 K (one-sigma) due to other forcings. If this temperature decrease is due to tropospheric aerosols a consequence would be that if emissions of these aerosols and their precursors were abruptly decreased, resulting in the forcing by these aerosols decreasing quite rapidly on account of their very short atmospheric residence time (about a week), GMST would be expected to rapidly increase to a new, higher value. While the possibility of such a hypothetical increase in GMST following cessation of emissions of tropospheric aerosols and precursor gases has been noted in previous studies [Andreae, 2005; Brasseur and Roeckner, 2005], neither study recognized the rapidity of the increase in GMST that would be expected. In contrast an abrupt decrease in emissions of CO₂ would result in only minimal decrease in GMST because of the long lifetime (ca 100 years) associated with excess atmospheric CO₂.

7. Discussion, conclusions, and implications

The findings of the present study may be considered surprising in several respects:

(1) The relatively small effective heat capacity of the global ocean that is coupled to the increase in global mean surface temperature over the five-decade period for which ocean heat content measurements are available, 14 ± 6 W yr m⁻² K⁻¹ (0.44 G J m⁻² K⁻¹), equivalent to about 150 m of the world ocean, and the correspondingly low effective planetary heat capacity \( C \), 16.7 ± 7.0 W yr m⁻² K⁻¹ (0.5 ± 0.2 G J m⁻² K⁻¹).

(2) The short relaxation time constant of global mean surface temperature in response to perturbations \( \tau \), 5 ± 1 years; and

(3) The low equilibrium climate sensitivity \( \lambda_s^{-1} \) inferred from (1) and (2) as \( \lambda_s^{-1} = \tau / C \), 0.30 ± 0.14 K/(W m⁻²), equivalent to equilibrium temperature increase for doubled CO₂ \( \Delta T_{x2} = 1.1 \pm 0.5 \) K. This value is well below current best estimates of this quantity, summarized in the Fourth Assessment Report of the IPCC [2007] to be "2 to 4.5 K with a best estimate of about 3 K and ... very unlikely to be less than 1.5 K".

This situation invites a scrutiny of the each of these findings for possible sources of error of interpretation in the present study.

Is the effective heat capacity that is coupled to the climate system, as determined from trends in ocean heat content and GMST, too low, or too high? For a given relaxation time constant \( \tau \), a lower value of \( C \) would result in a greater climate sensitivity, and vice versa. As noted above previous investigators have used similar considerations to suggest different values for \( C \), in one instance substantially greater than the value reported
here (20 - 50 W yr m⁻² K⁻¹) and in one instance with a range of a factor of 20, (3.2 - 65 W yr m⁻² K⁻¹) that encompasses the value determined here. Examination of Figure 4 suggests that it would be hard to justify a slope less than about 8 W yr m⁻² K⁻¹. Perhaps a more fundamental question has to do with the representativeness of the data that comprise the Levitus et al. [2005] compilation. In this context it might be noted that Willis et al. [2004] reported an heat uptake rate in the upper 750 meters of the ocean, based on satellite altimetry as well as in-situ measurements, of 0.86 ± 0.12 W m⁻², a factor of 7 greater than the Levitus et al. [2005] average for 1958-1995; a greater heat uptake rate would result in a greater effective ocean heat capacity and a lower climate sensitivity. However in a subsequent publication a year later Lyman et al. [2006] reported a rapid net loss of ocean heat for 2003-2005 that led those investigators to estimate the heat uptake rate for 1993-2005 as 0.33 ± 0.23 W m⁻², a value much more consistent with the long-term record in the Levitus et al. [2005] data set. The previous instances of several-year periods of net loss of heat from the ocean exhibited in the Levitus et al. [2005] data and shown in Figure 2 suggest the necessity of evaluating the effective heat capacity based on a long-term record.

Is the relaxation time constant of the climate system determined by autocorrelation analysis the pertinent time constant of the climate system? Of the several assumptions on which the present analysis rests, this would seem to invite the greatest scrutiny. A possible explanation for the short time constant inferred from the autocorrelation analysis might be that the autocorrelation is dominated by short term variability, such as that resulting from volcanic eruptions, and that the thermal signal from such a short perturbations would not be expected to penetrate substantially into the deep ocean. Two considerations would speak against such an explanation. First, the autocorrelation leading to the 5-year time constant extended out to lag times of 15 years or more with little indication of increased time constant for lag time greater than about 5-8 years (Figure 6). Also, recent studies with coupled ocean atmosphere GCMs have shown that the thermal signal from even a short-duration volcanic event is transported into the deep ocean and can persist for decades [Delworth et al., 2005; Gleckler et al., 2006 a,b]; such penetration of the thermal signal from a short-duration forcing would suggest that the autocorrelation of GMST over a decade or more would be representative of the longer time constant associated with the coupling to the deep ocean and not reflective simply of a short time constant associated with the ocean mixed layer.

Finally, as the present analysis rests on a simple single-compartment energy balance model, the question must inevitably arise whether the rather obdurate climate system might be amenable to determination of its key properties through empirical analysis based on such a simple model. In response to that question it might have to be said that it remains to be seen. In this context it is hoped that the present study might stimulate further work along these lines with more complex models. It might also prove valuable to apply the present analysis approach to the output of global climate models to ascertain the fidelity with which
these models reproduce "whole Earth" properties of the climate system such as are empirically determined here. Ultimately of course the climate models are essential to provide much more refined projections of climate change than would be available from the global mean quantities that result from an analysis of the present sort. Still it would seem that empirical examination of these global mean quantities – effective heat capacity, time constant, and sensitivity – can usefully constrain climate models and thereby help to identify means for improving the confidence in these models.

The empirical determinations presented here of global heat capacity and of the time constant of climate response to perturbations on the multidecadal time scale lead to a value of equilibrium global climate sensitivity of $0.30 \pm 0.14 \text{ K/(W m}^{-2}\text{)},$ where the uncertainty range denotes a one-sigma estimate. This sensitivity together with the increase in global mean surface temperature over the twentieth century would imply a total forcing $1.9 \pm 0.9 \text{ W m}^{-2};$ although the central value of this range is fairly close to the total greenhouse gas forcing over this time period, $2.2 \text{ W m}^{-2},$ this result is consistent with an additional forcing over the twentieth century of $-0.30 \pm 0.97 \text{ W m}^{-2}.$ The rather large uncertainty range could be consistent with either substantial cooling forcing ($-1.3 \text{ W m}^{-2}$) or substantial warming forcing ($+0.7 \text{ W m}^{-2}$), with aerosol forcing a likely major contributor. Because of the short response time of the climate system to perturbations, the climate system may be considered in near steady state to applied forcings and hence, within the linear forcing-response model, the change in temperature over a given time period may be apportioned to the several forcings. The estimated increase in GMST by well mixed greenhouse gases from preindustrial times to the present, $0.7 \pm 0.3 \text{ K};$ the upper end of this range approaches the threshold for "dangerous anthropogenic interference with the climate system," which is considered to be in the range 1 to 2 K [O'Neill and Oppenheimer, 2002; Hansen, 2004].

**Acknowledgment.** Supported by the U.S. Department of Energy's Atmospheric Science Program (Office of Science, OBER) under Contract No. DE-AC02-98CH10886.
References


Table 1. Empirical determination of effective global ocean heat capacity as ratio \((dH/dt)/(dT_s/dt)\), Method 1.

<table>
<thead>
<tr>
<th>Ocean depth m</th>
<th>(dT_s/dt) K yr(^{-1})</th>
<th>GISS</th>
<th>CRU</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>0.014 ± 0.002</td>
<td>0.012 ± 0.002</td>
<td>0.013 ± 0.002</td>
<td></td>
</tr>
<tr>
<td>700</td>
<td>0.139 ± 0.037</td>
<td>9.9 ± 3.0</td>
<td>11.6 ± 3.6</td>
<td>10.8 ± 4.0</td>
</tr>
<tr>
<td>3000</td>
<td>0.205 ± 0.062</td>
<td>14.6 ± 4.9</td>
<td>17.0 ± 5.9</td>
<td>15.9 ± 6.4</td>
</tr>
</tbody>
</table>

Row 2 gives \(dT_s/dt\) evaluated for GISS and CRU data compilations. Column 2 gives \(dH/dt\) evaluated from ocean heat content data compiled by Levitus et al. [2005] for ocean depths 300, 700, and 3000 m. Uncertainties in individual slopes are evaluated taking into account autocorrelation (Leith, 1973).

Table 2. Empirical determination of effective global ocean heat capacity from regression of ocean heat content anomaly \(H\) against global mean surface temperature \(T\), Method 2.

<table>
<thead>
<tr>
<th>Ocean depth m</th>
<th>Effective global ocean heat capacity W yr m(^{-2}) K(^{-1})</th>
<th>Average, Methods 1 &amp; 2 W yr m(^{-2}) K(^{-1})</th>
<th>Equivalent ocean depth m</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>6.0 ± 0.4</td>
<td>6.5 ± 0.8</td>
<td>69</td>
</tr>
<tr>
<td>700</td>
<td>9.4 ± 0.8</td>
<td>10.1 ± 1.3</td>
<td>110</td>
</tr>
<tr>
<td>3000</td>
<td>11.6 ± 1.4</td>
<td>12.2 ± 2.1</td>
<td>148</td>
</tr>
</tbody>
</table>

Regression slopes were obtained as the bisector of the slope of the ordinary least squares regression of \(H\) against \(T\) and the inverse of the slope of the regression of \(T\) against \(H\) [Isobe et al., 1990]; see Figure 4. Columns 2 and 3 gives \(C\) evaluated using the GISS and CRU temperature data, respectively, for ocean heat content data compiled by Levitus et al. [2005] for ocean depths 300, 700, and 3000 m. Uncertainties are evaluated as average deviation of \(C\) from the mean of values for the two temperature data sets; uncertainties in the average of Methods 1 and 2 encompass the range of estimates. Last column shows depth of the world ocean that would exhibit the same heat capacity as the effective heat capacity, evaluated for ocean fractional area of planet 0.71 and volume heat capacity taken as \(4.2 \times 10^6\) J m\(^{-3}\) K\(^{-1}\).
Table 3. Empirical determination of key properties of Earth’s climate system.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Unit</th>
<th>Value</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective global heat capacity $C$</td>
<td>W yr m$^{-2}$ K$^{-1}$</td>
<td>16.7</td>
<td>7</td>
</tr>
<tr>
<td>Effective global heat capacity $C$</td>
<td>GJ m$^{-2}$ K$^{-1}$</td>
<td>0.53</td>
<td>0.22</td>
</tr>
<tr>
<td>Effective climate system time constant $\tau$</td>
<td>yr</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>Equilibrium climate sensitivity $\lambda_s^{-1}$</td>
<td>K/(W m$^{-2}$)</td>
<td>0.30</td>
<td>0.14</td>
</tr>
<tr>
<td>Equilibrium temperature increase for doubled CO$<em>2$ $\Delta T</em>{2x}$</td>
<td>K</td>
<td>1.1</td>
<td>0.5</td>
</tr>
<tr>
<td>Increase in GMST over twentieth century $\Delta T_{s,20}$</td>
<td>K</td>
<td>0.57</td>
<td>0.085</td>
</tr>
<tr>
<td>Total forcing over twentieth century $F_{20}$</td>
<td>W m$^{-2}$</td>
<td>1.9</td>
<td>0.9</td>
</tr>
<tr>
<td>Lag in temperature change over twentieth century $\Delta T_{lag}$</td>
<td>K</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>Total greenhouse gas forcing over twentieth century $F_{G,20}$</td>
<td>W m$^{-2}$</td>
<td>2.2</td>
<td>0.3</td>
</tr>
<tr>
<td>Forcing in twentieth century other than greenhouse gas forcing $\Delta F_{20}$</td>
<td>W m$^{-2}$</td>
<td>-0.30</td>
<td>0.97</td>
</tr>
<tr>
<td>Temperature increase in twentieth century due to greenhouse gas forcing</td>
<td>K</td>
<td>0.66</td>
<td>0.32</td>
</tr>
<tr>
<td>Temperature increase in twentieth century due to CO$_2$ forcing</td>
<td>K</td>
<td>0.37</td>
<td>0.17</td>
</tr>
<tr>
<td>Temperature decrease in twentieth century due to other than greenhouse gas forcing</td>
<td>K</td>
<td>-0.09</td>
<td>0.29</td>
</tr>
<tr>
<td>Total forcing by well mixed greenhouse gases 1750-1998 $[\text{IPCC, 2001}]$</td>
<td>W m$^{-2}$</td>
<td>2.43</td>
<td>0.24</td>
</tr>
<tr>
<td>Temperature increase 1750-1998 due to greenhouse gas forcing</td>
<td>K</td>
<td>0.72</td>
<td>0.34</td>
</tr>
</tbody>
</table>
Figure 1. Estimates of equilibrium global climate sensitivity and associated uncertainty from major national and international assessments [Charney et al., 1979; IPCC, 2007; for citations to earlier IPCC reports see IPCC, 2007]. Equilibrium sensitivity given as increase in global mean surface temperature that would result from sustained doubling of atmospheric CO$_2$ (left axis) is converted to unit of K/(W m$^{-2}$) for global mean forcing of doubled CO$_2$ taken as 3.7 W m$^{-2}$ [IPCC, 2007].
Figure 2. Time series of global ocean heat content anomaly, left axis, and global mean surface temperature anomaly, right axis. Global ocean heat content data L300, L700, and L3000, are from Levitus et al. [2005] for ocean depths from the surface to 300, 700, and 3000 m, respectively. For L300 and L700 the data represent annual averages (1956-2002); for L3000, for which the measurements are more sparse, the data represent 5-year running averages (1958-1995) plotted at the center of the averaging period. CRU denotes global average surface temperature from the Climate Research Unit, University of East Anglia, U. K. [Jones et al., 2005]; GISS denotes global average surface temperature from the Goddard Institute for Space Studies, NASA, U. S. [Hansen et al., 1996]. Also shown are linear regressions to the several data sets.
Figure 3. Rate of increase of global heat content $dH/dt$ versus rate of increase of global mean surface temperature $dT_s/dt$ for 1960-2000; lines of constant heat capacity permit assessment of global heat capacity as ratio $(dH/dt)/(dT_s/dt)$. Values of $dH/dt$ are evaluated from data of Levitus et al. [2005] for ocean depths from the surface to 300, 700, and 3000 m, as in Figure 2. Values of $dT_s/dt$ are given as determined from the GISS and CRU compilations of temperature data as in Figure 2. Uncertainties are one-sigma estimates taking into account autocorrelation [Leith, 1973].
Figure 4. Graphs of ocean heat content data versus global mean surface temperature anomaly over the period 1956-2002. Global ocean heat content data L300, L700, and L3000, are from Levitus et al. [2005] for ocean depths from the surface to 300, 700, and 3000 m, respectively. a) Temperature anomaly data are from the GISS, left, and CRU, right, compilations. All data were adjusted to exhibit mean values of 0 to facilitate evaluation of the slopes, which were obtained as the bisector of the slope of the ordinary least squares regression of $H$ against $T$ and the inverse of the slope of the regression of $T$ against $H$ [Isobe et al., 1990]. Slopes, red, (upper left in each panel and Table 2) yield the global mean heat capacity in units of W yr m$^{-2}$ K$^{-1}$; uncertainty ranges, green, are calculated from estimated standard error of slope, evaluated as the square root of the estimated variance in the slope divided by the square root of the effective number of independent measurements taking into account autocorrelation [Leith, 1973]. Color bar indicates year of measurement; L3000 data represent 5-year running averages with the date at the center of the averaging period.
Figure 5. Autocorrelation analysis of time series data for global mean surface temperature, $T_s$. a) Original time series, GISS, [Hansen et al., 1996]; line represents linear regression. b) Normalized time series after detrending by linear regression in a. c) Autocorrelogram $r(\Delta t)$ of detrended data. d) Normalized residuals lagged by one year versus the normalized residuals; line represents linear regression. e) Normalized residuals after detrending by the lag-1 autocorrelation. f) Autocorrelogram of detrended residuals, showing no remaining autocorrelation. g) Relaxation time constant evaluated as $\tau(\Delta t) = -\Delta t / \ln r(\Delta t)$ up to the first non-positive value. Uncertainties on $r$ represent estimated standard deviation evaluated as the square root of the estimated variance of $r$ evaluated according to Bartlett [1946]; uncertainties on $\tau$ are standard error of estimate propagated from uncertainties on $r$. 
Figure 6. Relaxation time constants $\tau(\Delta t)$ as a function of lag time $\Delta t$ for various subsets of the GISS (left) and CRU (right) time series for global mean surface temperature; as in Figure 5g.
Figure 7. Relaxation time constants $\tau(\Delta t)$ (blue) and associated standard errors (red) as a function of lag time $\Delta t$ for the deseasonalized monthly GISS time series for global mean surface temperature; as in Figure 5g.