Chapter 1

Introduction

1.1 The El Niño-Southern Oscillation

1.1.1 El Niño: The Phenomenon

On the wall in the hall of one of this university's buildings are a sequence of images, dated from January, 1980 to February, 1998. The images are monthly averages of the vertical temperature profile of the equatorial Pacific Ocean. At both ends of this tableau, the images show warm water extending eastward to replace the normally cooler waters of the eastern Pacific. These are the El Niño events of 1982-83 and 1997-98, the strongest two of this century.

The news media, apparently lacking anything better to do, latched on to "El Niño" in the fall of 1997. El Niño received the blame for a litany of catastrophes: a major blizzard in Colorado, flooding in California, tornadoes in Florida, floods in coastal Peru, and a forest fire in Sumatra so severe, it blackened the skies, leading to a plane crash and closing airports as far away as Singapore. A commentator even made an "El Niño connection" to the financial crisis that hit Southeast Asia in late 1997. Of course, accompanying every mention of El Niño, the media shows a lack of understanding of what El Niño is.[1]

So what is El Niño? The name itself comes from a change in the ocean currents off the coast of Peru. There is an annual decrease in the westward currents off the Peruvian coast during December and January, accompanied by a warming of the ocean's surface temperature (Wyr75). The westward ocean current transports warm surface water to the western Pacific while also causing the upwelling of colder, subsurface waters in the east.

[1] Philander (Phi90) has a description of actual impacts of El Niño, specifically the one in 1982-83.

Riding that upwelling are nutrients that feed the fish population. When the currents slacken, the surface waters warm as upwelling decreases (Phi90). If this change is severe enough, the resulting loss of nutrients causes fish populations to decline. The Peruvian fisherfolk, noticing a connection between their catch, the warming of the waters, and the time of year, named the phenomenon "El Niño," after the Christ-child (Phi90).

At the beginning of this century, Sir Gilbert Walker noticed an interesting connection between the surface atmospheric pressure at various points across the Pacific Ocean (Bje69). Specifically, when the pressure in Darwin, Australia and other points in the western Pacific was high, the pressure in Tahiti and the central and eastern Pacific was low. The converse is also true. He termed this phenomenon the "Southern Oscillation" and related it to changes in rainfall and wind patterns throughout the tropical Pacific and Indian Oceans (Phi90). In the late 1960's, Bjerknes (Bje66, Bje69) made a connection between the Southern Oscillation in the atmosphere and the El Niño phenomenon in the ocean. The two are now known collectively as "ENSO," the El Niño-Southern Oscillation.

1.1.2 The Physics of ENSO

Bjerknes' key contribution to the field of ENSO physics was his proposal of a feedback mechanism connecting the atmosphere and the ocean. Figure 1.1 shows a schematic of that feedback loop. Consider first the presence of warm ocean waters on one side of an ocean basin and colder water on the opposite side. The warmer sea surface temperatures, or SST, generate rising motion. Additionally, warmer temperature decreases the vertical pressure gradient relative to a region of colder temperature. At some upper level, then, there will be a horizontal pressure gradient accompanied by a flow from the warmer end of the ocean to the colder end [Figure 1.1]. Sinking motion over the region of cold SST and another horizontal pressure gradient at the surface complete the circulation pattern. Consequently, there is a surface wind moving from the colder end of the ocean basin to the warmer one. In the Pacific Ocean, these are the easterly trade winds along the equator. Bjerknes termed the entire equatorial cell containing the trade winds the "Walker Circulation" (Bje69).

Let us return now to Figure 1.1 and consider the right side of the diagram to be the eastern end of an ocean basin, in analogy to the Pacific. The westward winds generate a westward surface ocean current, advecting warm water to the western end of the basin while

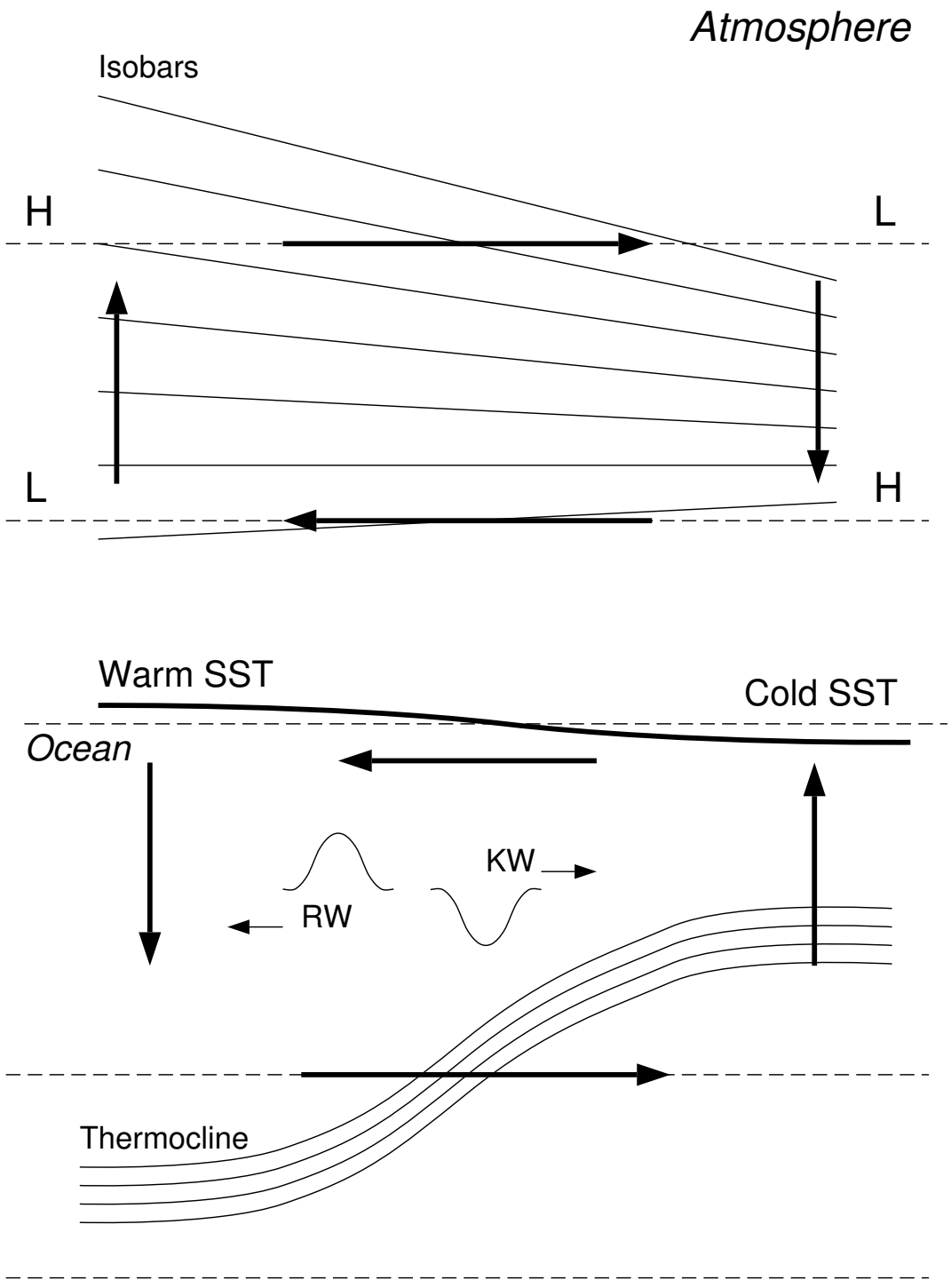

Figure 1.1: Schematic of the basic physics of ENSO. "H" and "L" mark high and low atmospheric pressure, respectively. "RW" and "KW" are abbreviations for "Rossby wave" and "Kelvin wave," respectively.

raising the sea level there. In the western Pacific, this generates the so-called "warm pool" along the equator and raises the sea surface an average of 0.7 meters (Bje66). As noted earlier, the surface current also generates upwelling and downwelling at the eastern and western boundaries of the ocean, respectively, by simple conservation of mass. The circulation is completed by a subsurface current along the thermocline, a steep temperature gradient separating warmer upper-ocean waters from the colder, abyssal waters (Bje69, Phi90). In the Pacific, the thermocline is about 50 meters thick, marking the boundary of an upper layer about 150 meters deep in the west, but surfacing in the east (Phi90). The current thus maintains a sea surface temperature gradient, the same gradient that, ultimately, generates the current. Thus, there is a positive feedback loop between the atmosphere and ocean (Bje66, Bje69).

Wyrtki (Wyr75) notes that a slackening of the surface wind removes the force maintaining the equatorial sea surface slope. He suggests that the resulting propagation of warm water eastward from the warm pool might take place in the form of an equatorial Kelvin wave. Cane and Zebiak (CZ85) comment that the westerly wind bursts common in the Pacific can also excite equatorial Kelvin waves. These Kelvin waves then depress the thermocline in the eastern Pacific, changing the Bjerknes feedback loop's direction. This would lead to an El Niño event. They also remark that ENSO takes place completely in the tropical Pacific. The Pacific Ocean, however, does not experience only El Niño events, but oscillates between two states, the El Niño or warm event, and the corresponding cold event, sometimes termed "La Niña" (Phi90).

In the late 1980's, Graham and White, as well as Schopf and Suarez (GW88,SS88b), published what is now considered the "canonical" description of ENSO physics. Both papers describe El Niño as only one phase of a complete oscillation, just as summer is a single phase of the annual cycle. The cycle begins with some perturbation to the thermocline in the central Pacific, where the system is most sensitive (SS88a, SS88b). The perturbation excites two waves within the thermocline: a westward-propagating equatorial Kelvin wave and an eastward-propagating equatorial Rossby wave. The two waves have opposite signs, i. e. a perturbation produces a downwelling Kelvin wave and an upwelling Rossby wave [or vice versa]. Let us begin with this scenario, indicated, incidentally, in Figure 1.1 by "RW" and "KW." Kelvin waves propagate across the Pacific basin in about 2 to 3 months,

suppressing the thermocline in the east (GW88). This alters the atmospheric circulation, creating more perturbations that generate downwelling Kelvin waves. This leads to the El Niño or warm phase of ENSO. Meanwhile, the corresponding upwelling Rossby waves have been propagating slowly towards the so-called maritime continent on a journey that takes about 9 months (GW88). These Rossby waves reflect off of Indonesia as upwelling Kelvin waves, which propagate rapidly towards South America. The upwelling Kelvin waves raise the thermocline, enhancing the atmospheric circulation which, in turn, generates more upwelling Kelvin waves, along with corresponding downwelling Rossby waves. The system thus enters the La Niña or cold phase of ENSO. The overall period of this oscillation is about 3-5 years (GW88).

In reality, the Kelvin waves reflect from the South American coast as Rossby waves with the same sign. There is, then, an entire train of Kelvin and Rossby waves, each, in sequence, elevating or lowering the thermocline, then reflecting off of the basin boundaries. Naturally, these wave trains dampen out over time, especially upon reflection.

On a final note, there are broader impacts of this oscillation. Due to its effects on atmospheric circulation, the cold phase of ENSO will enhance convection over Indonesia, leading to wetter conditions, while causing drier conditions over coastal South America (RC82). During a warm event, the opposite is true. Precipitation in Indonesia is weaker than average and stronger in coastal South America. During severe warm events, this can manifest itself in severe flooding in Peru and Ecuador.[2] A severe warm event also has connections to weak or absent monsoons in India (SP83, Phi90).

1.1.3 Modeling Efforts

Due to its strong human impact, there is much interest in modeling and forecasting ENSO. To describe ENSO behavior in a compact form, researchers primarily use two indices. The first, the Southern Oscillation Index [hereafter SOI], measures anomalous pressure differences between Darwin, Australia and Tahiti. The second type of index is a mean sea surface temperature [SST] of a predefined region in the equatorial Pacific Ocean, the most commonly used being the SST in the so-called "NINO3" region. This thesis makes heavy

[2] I have seen hypotheses that the ancient Moche civilization of Peru was wiped out by a series of strong El Niño events. Severe flooding wiped out crops, while the cessation of coastal upwelling reduced fish catches.

use of both the SOI and NINO3 SST as representatives of ENSO behavior. I shall return to them in greater detail in chapter 5.

One of the most frequently used models, for forecasts and as a basis for other models, is the two-layer model of Cane and Zebiak (ZC87, hereafter referred to as the CZ model). Cane et. al. (CMZ90, CMZ91) and Bürger et. al. (BZC95a, BZC95b) both analyze the CZ model in detail. Battisti (Bat88) builds upon the CZ model, and later, with Hirst (BH89), linearizes it. They use the linearized version of CZ to derive the simple delay equation model of ENSO first proposed by Suarez and Schopf (SS88b). This model of ENSO as a delay equation, a well-studied class of ordinary differential equations, will return in later discussion.

In addition to the CZ model, several researchers have used a full global circulation model [GCM] to study ENSO (Mee90, BLKR91). GCM's are highly complex, modeling not only the tropical Pacific, but the whole Earth. Consequently, there exist several "intermediate" models, hybrids that do not make some of the simplifying assumptions of the CZ model yet do not attempt to describe the entire globe (Nee90, Kle91, JN93a, JN93b, JN93c). At the other extreme is the model of Vallis (Val88). This highly simplified two-layer model, cast into a form similar to the famous "Lorenz butterfly" model (Lor63), will be used in the work of chapter 6.

1.1.4 Predictability and the "Spring Barrier"

One of the key aspects of forecasting and model analysis is the study of how well a particular model or forecasting technique predicts the behavior of nature. In addition to examining the predictability of models and algorithms, one can also examine the "inherent" predictability of a dataset using lagged autocorrelations. Troup (Tro65) and Wright (Wri79, Wri85) did such an examination, using the SOI and a form of lag autocorrelation called persistence. As defined in their works, persistence is a cyclostationary statistic. Cyclostationary statistics, the appropriate statistics to use when a dataset has a natural periodic cycle (Gla61, HN96), are always a function of an initial — i. e. leading — phase of the natural periodic cycle in the data. Thus, cyclostationary statistics of data with an annual cycle could be a function of seasons, months, days, etc. The persistence, being a lag autocorrelation, is a function of both the initial phase and, of course, a lag.

Figures 1.2 and 1.3 contain "persistence maps" of the SOI and the SST in the NINO3 region of the Pacific ocean, respectively. The maps are similar to ones shown by Wright (Wri79, Wri85) and Torrence and Webster (TW98). Note the rapid decrease in persistence during the boreal spring, at a leading phase of 0.25-0.5 yrs. Webster and Yang (WY92) termed this decrease a "predictability barrier." Torrence and Webster (TW98) suggest that this "barrier" occurs as the the climate system makes a transition from one state to another during the boreal spring. Xue et. al. (XCZB94) additionally show that the barrier is associated with low variance in the data during the spring.

The initial suggestion by Webster and Yang (WY92) that the "barrier" may indicate an inherent limit to ENSO predictability caused some controversy. There has been work suggesting that this decrease in persistence may not necessarily hinder forecasting (CZBC95). Some researchers may have questioned the physicality of this "predictability" barrier. Could it not also be an artifact of the statistics? Before one can answer that question, however, one needs to better understand the properties of cyclostationary statistics in general. The presence of such a shape, hereafter called the "persistence barrier" or simply "barrier," could indicate some interesting underlying behavior.

Although it is not yet clear how the barrier relates to the ability to forecast ENSO, it is nonetheless a statistical property of the system. As such, it warrants further investigation, for deeper understanding of persistence barriers may provide further insight into the causes and dynamics of ENSO. There may even be a connection between persistence barriers and other measures of predictability. Even if there is not such a connection, statistical features of a system can still provide useful benchmarks for comparing models with data. We shall return to these issues more fully in chapters 4-5.

1.2 Chaos Strikes El Niño

1.2.1 About Chaos

The term "chaos" is used to describe any system with aperiodic, irregular behavior that additionally contains no stochastic forcing. Often, the equations of motion for such a system are known exactly, even though the behavior is irregular. Much of the modern research into this field began with the fundamental work of Lorenz (Lor63), who examined

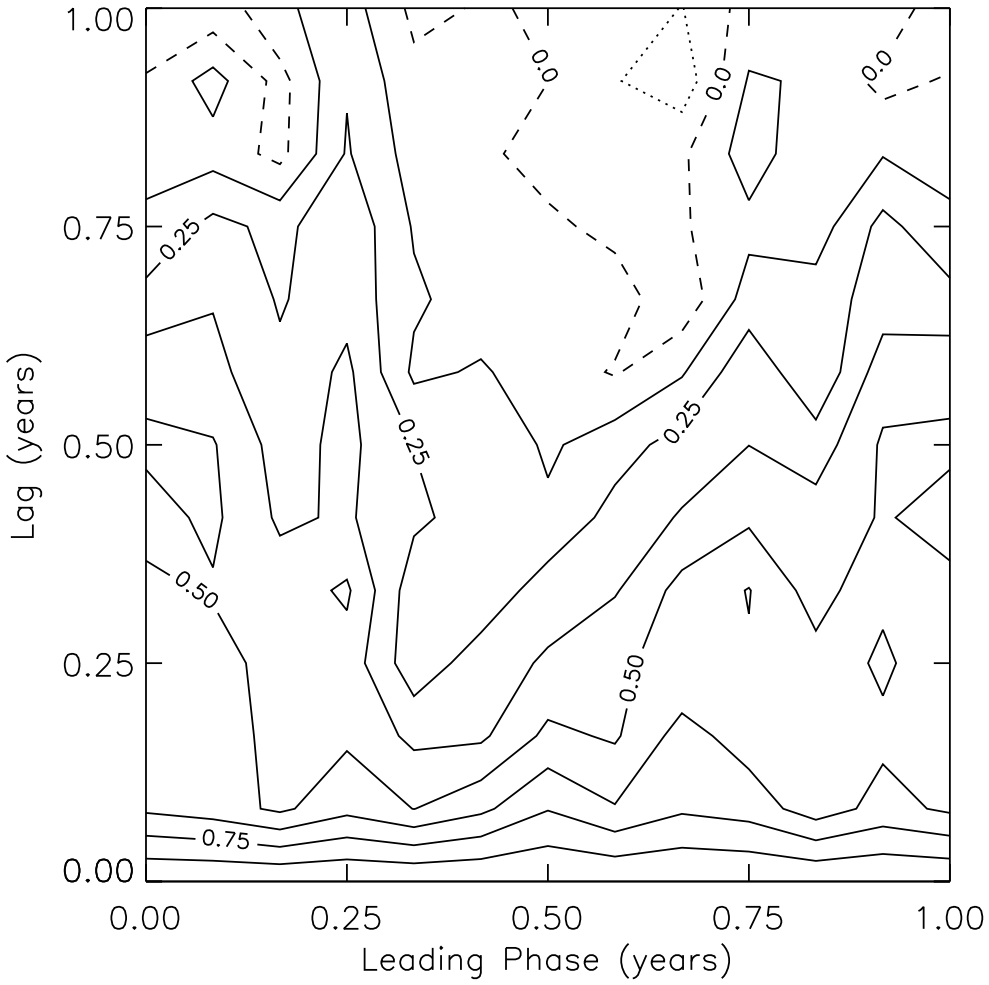

Figure 1.2: Persistence map of the SOI. Contour intervals are in steps of $\frac{1}{8}$. Solid contours denote positive values of persistence, while dotted contours denote negative values. The zero-contour is dashed.

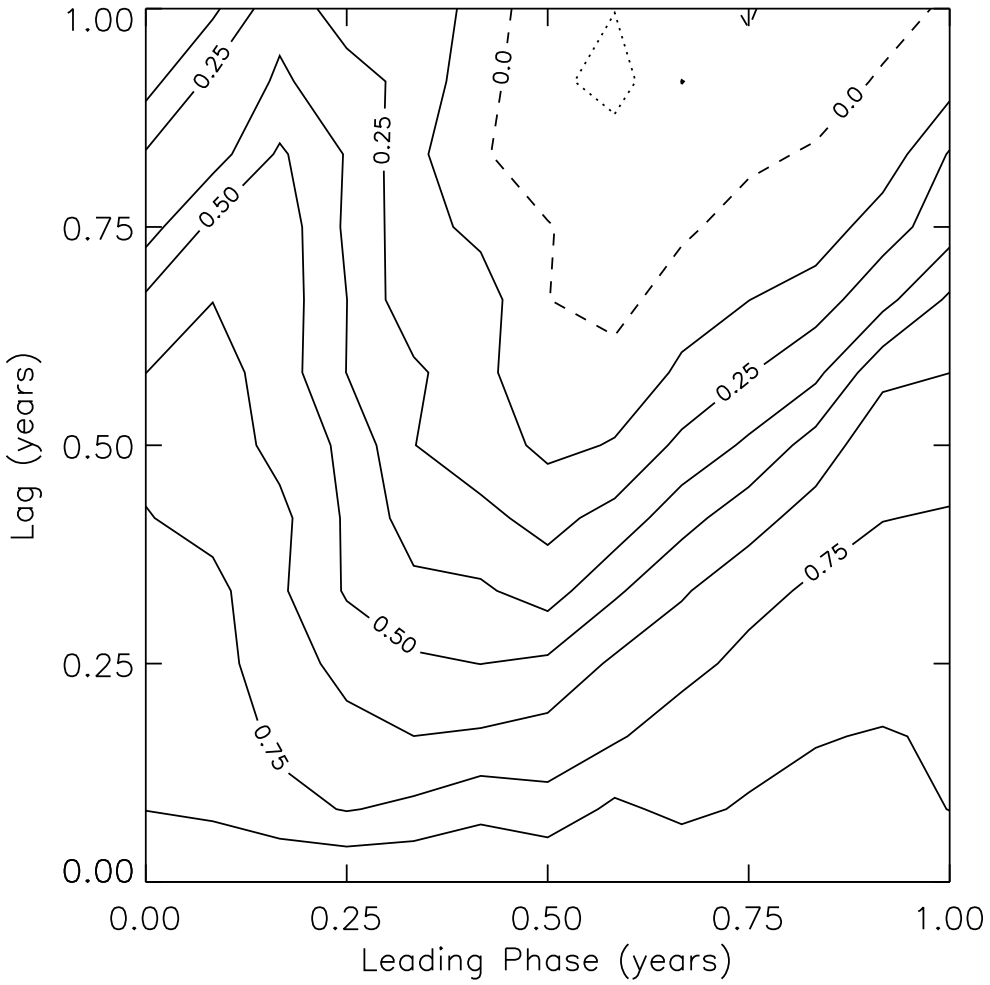

Figure 1.3: Persistence map of the NINO3 SST. The contours are plotted in the same fashion as in Figure 1.2.

the model:

$$\dot{x} = \sigma(y - x), \tag{1.1}$$

$$\dot{y} = -xz + rx - y, \tag{1.2}$$

$$\dot{z} = xy - bz, \tag{1.3}$$

a simplified, truncated model of atmospheric convection. These equations are now well-known as the "Lorenz equation" or the "Lorenz butterfly." Lorenz (Lor63) found that these equations exhibited aperiodic behavior and high sensitivity to initial conditions. That is, even tiny errors in an initial condition can cause drastic changes in the behavior of the system.

Zeng et. al. (ZPE93) published a good summary of the field of "chaos theory," more formally known as nonlinear dynamics, and its applications to the atmospheric sciences. They describe the early work of Poincaré at the beginning of the century, work not examined again until the 1960's. They discuss the definition of a "strange attractor" by Ruelle and Takens, and of the various routes to chaos a dynamical system can take. The general description of the theory of dynamical systems that follows comes from both Zeng et. al. (ZPE93) and Tabor (Tab89).

Consider some physical system, containing many variables that depend on time and, possibly, spatial location. In the theory of dynamical systems, each time-dependent variable of the system constitutes a dimension of a linear vector space, known as "phase space." If the system also depends on location, then the dimensions of phase space are all of the variables evaluated at all possible positions. As the system evolves through time, it traces out a trajectory through phase space. Sometimes, a system's trajectory may stay in one specific region of phase space. That region is termed an "attractor."

It can be very difficult to think about trajectories through phase space with a large number of dimensions. Poincaré thought of a simpler way to envision phase space. Consider a 2-dimensional "slice" of a 3-dimensional phase space, intersecting the trajectory. It may intersect a fixed region of phase space, or it may intersect the trajectory at "regular intervals" in phase space. The intersection points create an image, the so-called "Poincaré Section."[3]

[3]In general, a Poincaré Section is an $(N-1)$-dimensional subspace of an N-dimensional phase space.

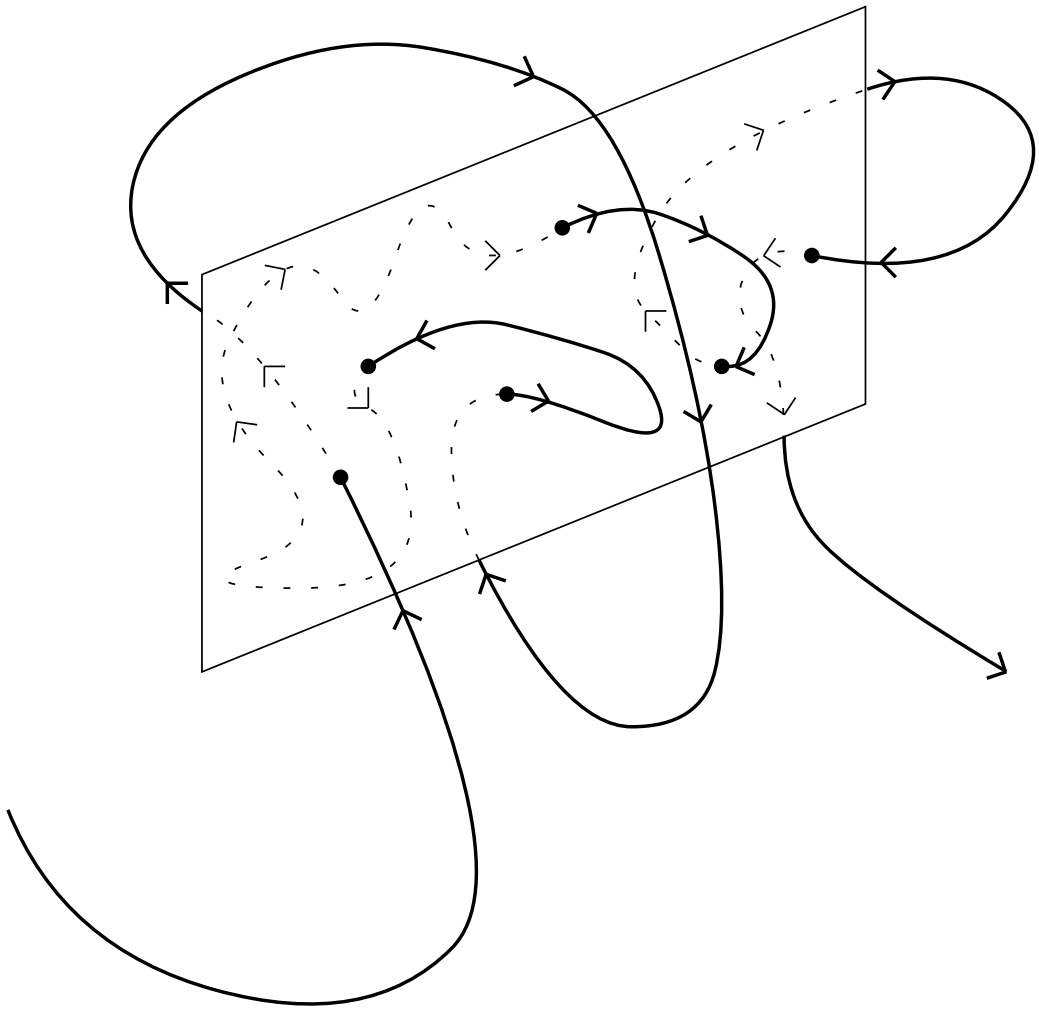

Figure 1.4: Schematic of a Poincaré Section for a 3-dimensional phase space, taken at a fixed location.

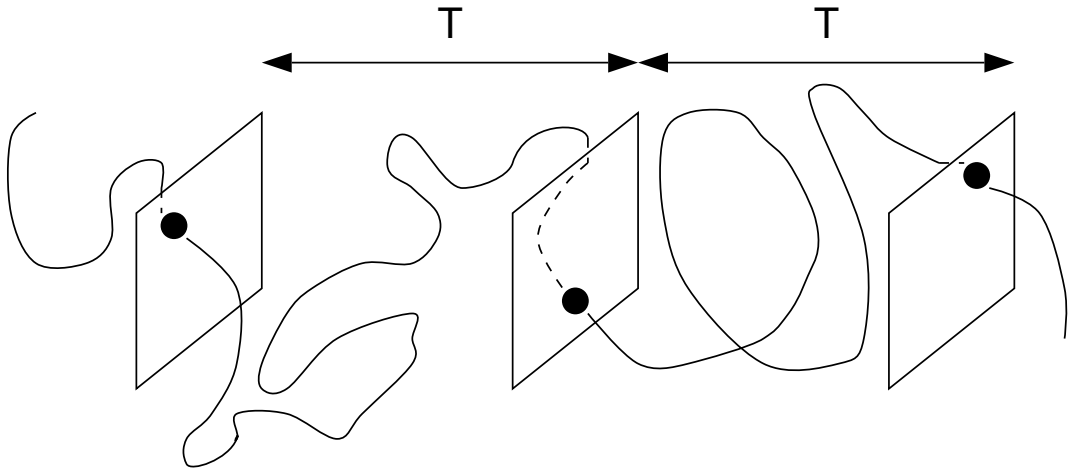

Figure 1.5: Schematic of a Poincaré Section for a 3-dimensional phase space, taken at regular intervals, "T." Note that "T" is not necessarily a time; it may be a distance in phase space.

The Poincaré Section of a periodic system shows an attractor composed of a finite set of points. Other attractors appear as circles or other simple, closed curves. The Poincaré Section of a strange-attractor, however, exhibits a complex, folded structure reminiscent of a fractal.[4]

Portraits of phase space are not the entire story of dynamical systems theory. The equations of motion for a system may contain parameters, independent of time, that describe physical properties of the system, such as damping coefficients, coupling strengths, driving forces, etc. Altering these parameters alters the behavior of the system. One can envision the possible values of every parameter also forming a space, a "parameter space," if you will. Different points in parameter space correspond to different behaviors of the system, i. e. different attractors and trajectories in phase space. It is not uncommon to examine how the attractor in phase space changes as one alters the parameters "along a path" through parameter space. Such a path can describe a "route to chaos."

To elucidate the previous paragraph, consider the following example. Suppose that a system has two different attractors in its phase space. Suppose, too, that one of the system's parameters, say, a damping coefficient, changes the distance between the two attractors. As one alters the damping coefficient, the two attractors eventually overlap. The system now alternates between the two attractors in an aperiodic fashion; the system has become chaotic. The scenario described here is also known as a "crisis" and constitutes a route to chaos.

Another route to chaos, related to the aforementioned "crisis," is the Pommeau-Manneville route. Under this route to chaos, a single attractor splits into two overlapping ones. Both of these types of chaotic systems are known as "intermittent chaos," for the system will exhibit intervals of periodic behavior punctuated by irregular behavior. Another route is the so-called period-doubling bifurcation. As one changes a parameter of the system undergoing periodic motion, the period doubles repeatedly. After going from twice its original period to four times its original period and so on, the system eventually becomes aperiodic, i. e. chaotic. The interested reader can read about these and other routes to chaos in Zeng et. al. (ZPE93) Tabor (Tab89), and the sources they reference.

There is another route to chaos, the quasi-periodic route to chaos. To simplify the discussion, consider a simple system with only two parameters. Suppose that, as one changes

[4]See Tabor (Tab89) for a complete explanation of why this is so.

the parameters, the period of the system remains fixed at a particular value, then "jumps" to a new value, at which it remains fixed. This behavior is known as "mode locking," and a one-dimensional plot of the system's period as a function of one parameter forms a "Devil's Staircase" (JBB84, BBJ84, Bak86). Between any two mode-locked "steps" of the staircase, there are an infinite number of other steps, hence the name "Devil's Staircase." If one were to plot the system's period as a function of all parameters, the mode-locked regions would form "petals," often referred to as "Arnol'd Tongues" or a "Devil's Flower" (Bak86).

Between the tongues, there are regions of non-mode-locked behavior. The system undergoes regular motion, but in a fashion that is "quasiperiodic." Quasiperiodic behavior is not chaotic; it is not irregular, but it is non-repeating. The function $f(t) = \cos(\pi t) + \cos(11t)$ is quasiperiodic. It comes close to repeating its previous values, but never does so exactly. Any function, in fact, that contains the sum of two other functions the ratio of whose frequencies forms an irrational number is quasiperiodic.

Along a particular, "critical" curve through parameter space, the tongues meet. That is, the system experiences mode locking, regardless the parameter values, and the Devil's Staircase is said to be "complete" (JBB84, BBJ84, Bak86). Beyond the critical curve, the tongues "overlap." The system, faced with being in two different mode-locked states simultaneously, alternates between the two in an irregular fashion.

The key point to the previous paragraph is the idea of a "critical curve" — or even a critical surface — in parameter space. The critical curve is simply a specific set of parameter values. Parameters to one side of the critical curve make the system periodic or quasiperiodic. Parameters on the other side produce chaos. At parameter values along the critical curve, the periods of the system collectively form the "complete Devil's Staircase," periodic, mode locked steps between which lie an infinity of other mode locked steps. In fact, this "infinity of steps" has a fractal structure to it (JBB84, BBJ84). The presence of a Devil's Staircase is a hallmark of the quasiperiodic-route to chaos.

1.2.2 El Niño on the Devil's Staircase

Surprisingly, several researchers have found examples of the quasiperiodic route to chaos in models of ENSO. Tziperman et. al. (TSCJ94) used the delay-equation model of Schopf and Suarez (SS88b), adding to it a periodic forcing term. They found transitions from

a stable periodic state to a quasiperiodic regime. Tuning the model's parameters further led to frequency-locked behavior, followed by chaos. Jin et. al. (JNG94) used the more complex Jin-Neelin model (JN93a, JN93b, JN93c), a set of four coupled partial differential equations. Using this model, they constructed a Devil's Staircase using two of the model's parameters.

Because a delay equation is a fairly simple system, one may wonder if the quasiperiodic chaos seen by Tziperman et. al. (TSCJ94) isn't actually a property of the model alone. The work of Jin et. al. (JNG94) would seem to indicate otherwise. Tziperman, however, provides a more convincing example. In a later paper, Tziperman and colleagues (TCZ95) perform computations using the full CZ forecast model (ZC87). Using the equivalent of the SST in the NINO3 region of the model, they perform a phase space reconstruction to examine the frequency behavior of the model's output (TCZ95). First, they see chaos even when the CZ model contains no seasonal forcing. Second, they demonstrate a quasiperiodic route to chaos in the model.

Other researchers have examined examples of chaos in other ENSO models. Chang et. al. (CWLJ94) also use a complicated ENSO model. They find a sequence of period-doubling bifurcations leading to chaos. Wang and Fang do something more interesting (WW96). They derive an ENSO model from "first principles," then simplify it to discern the "essential" dynamical system. Upon this system, they perform a "stability analysis" [see chapter 6 or section B.2 for more information about stability analysis] and compare it to data to verify that it does indeed capture the "basic dynamics" observed in nature. In addition to finding periodic and chaotic behavior, they also find the largest growth of perturbations in the boreal spring, an intriguing connection to the "spring barrier" described in section 1.1.4.

1.2.3 Noisy Niños

A major issue is missing from the previous discussion of chaotic behavior and ENSO. A chaotic system is distinguishable from a stochastic system only when the chaotic behavior is "low-order," i. e. the system's chaotic attractor is an object with few dimensions (Wei). Clearly, the Pacific Ocean and the atmosphere above it do not constitute a low-dimensional system! For this reason, some researchers feel that ENSO is driven by noise. Such works are not the focus of this dissertation and are presented as examples of an alternate viewpoint.

Balmaseda et. al. (BAD94) use an ENSO model with stochastic atmospheric forcing

based on data. Using a linear stochastic model, Xue et. al. (XCZB94) examine issues of predictability, with connections to the persistence barrier of section 1.1.4. Penland and Magorian (PM93) and Penland and Matrosova (PM94) develop an ENSO forecast technique by applying linear inverse modeling to another linear stochastic model of ENSO. With this model, Penland and Sardeshmukh (PS95) find a forecast range of 15 months. The interested reader should also examine the work of Blumenthal (Blu91), who develops some techniques used by these stochastic modeling efforts.

1.3 Stability, Instability, and Predictability

1.3.1 Lyapunov Exponents

There are other ways to describe predictability besides correlations and the forecast skill of models. Lorenz (Lor65) stated three possibilities for the lack of forecast skill in weather predictions:

(1) The system is not deterministic.

(2) Observations used to initialize the models are not accurate enough.

(3) The models and forecast techniques do not describe the system adequately enough.

Correlations can assist the modeler to discern how well his models match the behavior of the actual physical system. The first case, the presence of stochastic forcing, certainly occurs in nature at the quantum level. While predictability behavior at small scales has some effect on larger scale dynamics (Lor69), it is not clear how much, if any, of the quantum-level random forcing propagates to macroscopic scales.

In 1969, Lorenz described the predictability of a system based on the second aforementioned case, namely the effects of errors in initial conditions (Lor69). He formulated three types of predictability. In systems of the first type, initial errors either remain constant or shrink over time. Such systems, e. g. periodic systems, are predictable for all time. Systems of the second and third types exhibit error growth, and are therefore predictable over only a finite length of time. If one can always extend this period of predictability by shrinking the size of the errors, the system fulfills Lorenz' second type of predictability. For the third type of predictability, improving the accuracy of the initial conditions eventually

ceases to have an effect on the predictability of the system. The period of time over which the system is predictable is inherently fixed by the dynamics.

Since Lorenz's descriptions of predictability and its relation to error growth, forecaster often look for "optimal perturbations" in their models (TL95, MK96). These optimal perturbations are directions in the model's phase space, directions in which errors grow the most rapidly. They bear some relation to another quantity, the Lyapunov exponents. Consider a volume of perturbations to a trajectory through phase space, as shown in Figure 1.6. As the system evolves, the volume of perturbations distorts, growing in some directions and contracting in others. The Lyapunov exponents measure the exponential rates of growth and contraction of this volume.

The Lyapunov exponents actually come in three varieties. When the exponents measure growth rates of volumes over the whole of a strange attractor, they are termed "global" (BGGS80a, amongst numerous others). One of the definitions of a "chaotic system" is a system which has at least one positive nonzero Lyapunov exponent. That is, the system experiences exponential error growth.

There is also a version that measures these divergence rates locally over a finite region of the attractor (Nes89). These are of great use in real systems, for which one only needs the error growth behavior during, say, the next week, not until the end of the universe. One can also define "instantaneous" Lyapunov exponents that measure divergence rates "at a point" in phase space (VGR90). I shall return to all of these in great mathematical detail in chapter 2.

1.3.2 Applications to the Atmospheric Sciences

The concept of Lyapunov exponents has numerous applications within the atmospheric sciences. As noted in the previous section, forecasters often use the so-called optimal perturbations to determine the error growth characteristics of their forecast models. These optimal perturbations are, in fact, the so-called "Lyapunov eigenvectors" (MK96) defined later in chapter 2. Additionally, Trevisan and Legnani (TL95) show that measures of error growth based on the optimal perturbations converge to the Lyapunov exponents. Additionally, the optimal perturbations are related to techniques used to reconstruct the phase space of a system from data, specifically the so-called principle-component analysis (ZPE93).

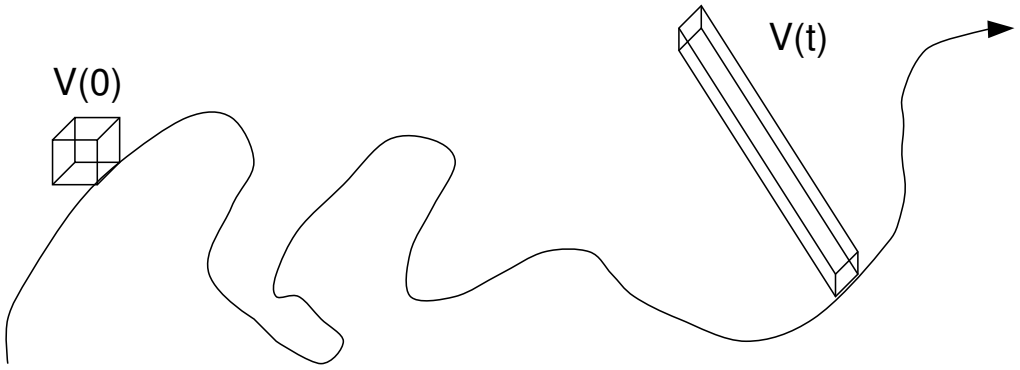

Figure 1.6: Distortion of a volume of perturbations, $V\,(0)$, as it evolves through phase space.

Principle-component analysis is also known as a singular value decomposition or an empirical-orthogonal function analysis. Zeng et. al. (ZPE93) discuss this in greater detail.

Moore and Kleeman (MK96) perform an extensive analysis of the optimal perturbations of the Kleeman model (Kle91). They find a connection between the optimal perturbations and the onset of ENSO events. They also note that the largest error growth for the Kleeman model occurs during the boreal spring, the time of the predictability barrier of section 1.1.4. In a similar vein, Yoden and Nomura (YN93) discuss the relationship between Lyapunov exponents and optimal error growth. They then apply the exponents to such problems in fluid dynamics as zonal flow and Rossby waves.

Vannitsem and Nicolis (VN94) examine the Lyapunov exponents of a thermal convective model. The model is a set of partial differential equations evaluated at over 100 spatial grid points. Consequently, the phase space for this system has a rather large dimension, yielding a correspondingly large number of Lyapunov exponents. That they compute the Lyapunov exponents for such a high-dimensional system is itself rather impressive. Using the exponents, they find that error growth and predictability are due to chaos from several interacting spatial scales. Instabilities at small scales generate chaos while compounding the instabilities at larger scales. Their description of this interaction has self-similar, almost fractal qualities, a fascinating concept.

More closely related to the work of this thesis is the paper by Lepri and colleagues (LGPA93). They examine the Lyapunov exponents of delay equations. Recall from section 1.2.2 that Tziperman et. al. (TSCJ94) used a delay equation model of ENSO to examine manifestations of low-order chaos in ENSO. Lepri et. al. (LGPA93) note that most of the Lyapunov exponents scale as the inverse of the delay time. They also reveal certain special cases in which the largest Lyapunov exponent does not scale. This is an intriguing result: the magnitude of the Lyapunov exponents, the descriptors of error growth for the system, is a function of a time scale of the delay equations, the delay time.

Abarbanel and Lall (AL96) use Lyapunov exponents to examine the predictability timescales of the volume of the Great Salt Lake. The perform a phase space reconstruction on a 150-year data set using the delay method proposed by Takens.[5] The data set contains about 3600 points total. Nevertheless, Abarbanel and Lall are able to perform a reasonable

[5] Zeng et. al. (ZPE93) also discuss this method.

phase space reconstruction. Although some attempts at phase space reconstruction fail unless the dataset contains a vast number of points, Abarbanel and Lall note that the number of data points is not nearly as important as how well the data samples the attractor. In their case, the 3600 measurements of volume changes in the Great Salt Lake create a well-formed attractor (AL96).

The previous paragraph contains a highly important point. There have been numerous attempts to compute measures of chaos from atmospheric and oceanic data, especially fractal dimension (Lor91, ZPE93). Based on these measures, one would get the impression that the entire Earth system was a low-dimensional chaotic system. Lorenz (Lor91) demonstrated why all of these computations of fractal dimensions were vastly underestimated. He examined a system comprised of numerous coupled chaotic oscillators, most of which were weakly coupled to one another. The phase space of this system thus contains a low-dimensional subspace in which coupling is strong, i. e. there is a locally strong fixed point in a region of the attractor. Lorenz integrated this system and estimated the fractal dimension from one of its variables. His results show that some variables produce estimates of dimension much smaller that the true size of the phase space. There are some variables, however, that nevertheless yield a reasonable estimate (Lor91).

The implications for ENSO are immediately clear. Perhaps these manifestations of low-dimensional chaos in the models are only local. That is, in the vast phase space of the Pacific atmosphere-ocean system, there is some low-dimensional region of the attractor containing a strong fixed point. Lorenz feels that this is the case for examples of low-order chaos in the Earth system and urges caution (Lor91). One should not immediately discount such manifestations as nonphysical, but should examine them from a new perspective, as descriptors of local behavior.

1.4 Motivation

Thus far, we have seen how models of the El Niño-Southern Oscillation exhibit features of low-dimensional chaos. As revealed in the previous section, such manifestations of low-dimensional behavior in high-dimensional dynamical system may indicate localized behavior on the system's strange attractor. Despite the numerous papers discussing low-

order chaos in ENSO models, none have yet made any firm connections to physical data. The only connections made are statements of how well a given set of equations model ENSO data. It would certainly be more satisfying if some form of data analysis revealed low-dimensional behavior in the Southern Oscillation Index [SOI] or the sea surface temperature in the "NINO3" region of the equatorial Pacific Ocean [NINO3 SST].

Section 1.1.4 discussed the so-called "predictability barrier" seen in the persistence of both the SOI and NINO3 SST. However, there is as yet no connection between persistence and predictability as defined in a dynamical systems sense, i. e. the Lyapunov exponents. In fact, no one appears to have examined the properties of the statistic termed "persistence." A barrier-like feature in the persistence of a time series, a feature such as those seen in Figures 1.2 and 1.3, may indicate interesting dynamics. Furthermore, those papers that attempt to prove or disprove the presence or absence of a barrier (XCZB94,CZBC95,BDA95, TW98) use only visual comparisons of persistence. A more quantitative, less subjective measure of barrier-like features in persistence would be preferable.

These ideas raise a question: Could the "barriers" in persistence somehow be connected to low-order chaotic behavior? Such a connection would neatly connect the "predictability barrier" to dynamical systems behavior while simultaneously providing a data-based manifestation of low-dimensional chaos. Making some progress towards answering this question is the focus of this research.

I shall begin this research with two background chapters. Chapter 2 examines the Lyapunov exponents, including the global, finite-time, and instantaneous exponent spectra, as well as describing the Lyapunov eigenvectors. A secondary purpose of this chapter is a unified summary of the numerous, nontrivial papers that originally defined these quantities. In chapter 3, we will examine issues surrounding the fitting of data to nonlinear models. Section 3.2 will discuss genetic algorithms and their uses with other nonlinear optimization techniques. Section 3.3 provides a definition of a correlation-like quantity and confidence intervals as applied to the problem of nonlinear least-squares fitting.

Chapter 4 begins the primary focus of the thesis with a detailed analysis of persistence. Section 4.2 presents a formal definition of cyclostationary statistics, the class of statistics to which persistence belongs. Sections 4.2.2-4.2.5 then calculate the persistence for a progression of simple but increasingly complex time series. Not all time series have a

persistence that resembles the "persistence barrier" seen in ENSO data. Using the results of these sections, I shall describe a set of criteria that a timeseries must fulfill in order to produce persistence "barrier." Section 4.3 defines a model containing the key features of the shape known as the "barrier." Henceforth, I shall use the term "persistence barrier" or "barrier" to describe this shape. A nonlinear least-squares fit to this model provides a quantitative measure of the properties of persistence barriers in data.

In chapter 5, I will apply the results of chapter 4 to the El Niño-Southern Oscillation.[6] Section 5.2.1 uses the aforementioned measure to quantify the barriers in the NINO3 SST and the SOI. Section 5.2.2 numerically examines the conditions for producing a barrier, revealing a connection between persistence barriers and phase-locking to the annual cycle. Finally, in section 5.3, I use the measure of barrier properties to study the interdecadal variability of ENSO.

The connection between persistence barriers and phase-locking to the annual cycle is tantalizingly similar to the quasiperiodic route to chaos in systems such as the damped forced pendulum (JBB84, BBJ84). Because a damped forced pendulum is the simplest physical system exhibiting the quasiperiodic route to chaos, chapter 6 begins with a detailed description of this system. We shall learn that the pendulum contains other routes to chaos, such as period-doubling and intermittency. As noted in section 1.2.2, ENSO models also exhibit such routes to chaos. I shall also examine the ENSO model of Vallis (Val88), a version of the Lorenz equations [Eqs. (1.1-1.3)] with periodic forcing. The chapter concludes with an attempt to connect the instantaneous Lyapunov exponents to persistence using these two dynamical systems.

[6]The material in chapters 4 and 5 has been submitted to the Journal of the Atmospheric Sciences (WW97a, WW97b). The papers were co-authored with Dr. Jeffrey B. Weiss.