13.1 – Functions of Many Variables

In $\mathbb{R}^2$, we had y as a function of one variable x.

- $y = f(x)$ is the set of points (x, y) that satisfy f.

- x = independent variable (input) Domain
- y = dependent variable (output) Range

In $\mathbb{R}^3$, z is a function of two variables x and y

- $z = f(x, y)$ is the set of all triples (x, y, z) that satisfy f.
- x and y are both independent (x, y) = domain
- z is dependent z = range

Evaluating Functions of two variables:

#6) $f(x, y) = 4 - x^2 - 4y^2$ (or $z = 4 - x^2 - 4y^2$)

c) $f(2, 3) = 4 - (2)^2 - 4(3)^2 = 4 - 4 - 4(9) = -36$

Triple: $(2, 3, -36)$

#8) $g(x, y) = \ln|x + y|$

c) $g(e, 0) = \ln|e + 0| = \ln|e| = 1$

Triple: $(e, 0, 1)$

#16) $f(x, y) = 3xy + y^2$

a) $\dfrac{f(x + \Delta x, y) - f(x, y)}{\Delta x}$

$f(x + \Delta x, y) = 3(x + \Delta x)y + y^2$

$= 3xy + 3\Delta xy + y^2$

$\dfrac{(3xy + 3\Delta xy + y^2) - (3xy + y^2)}{\Delta x}$

$= \dfrac{3\Delta xy}{\Delta x} = \boxed{3y} \rightarrow \left(\dfrac{d}{dx} f(x, y)\right)$

Determining whether z is a function of x and y:

(2) $xz^2 + 2xy - y^2 = 4$

$$z = \pm\sqrt{\frac{4 - 2xy + y^2}{x}}$$

recall "for every x value there must be at most one y value"

we have 2 z values for every xy pair...

this fails the vertical line test and thus

is **not a function**

Finding the Domain:

(18) $f(x,y) = \sqrt{4 - x^2 - 4y^2}$

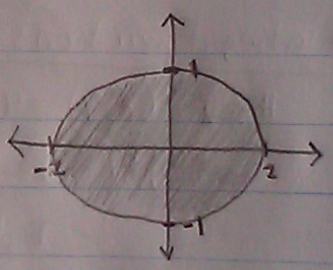
even root, so these must be positive or zero

$4 - x^2 - 4y^2 \geq 0$

$-x^2 - 4y^2 \geq -4$

$x^2 + 4y^2 \leq 4$

$\frac{x^2}{4} + \frac{y^2}{1} \leq 4$



(use a dotted line for < or >)

(22) $f(x,y) = \ln(xy - 6)$

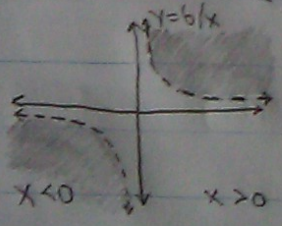
$xy - 6 > 0$ (since $\ln(a \leq 0)$ is bad)

$xy > 6$

neither x or y can ever be zero...

$y > \frac{6}{x}$ (when x is positive)

$y < \frac{6}{x}$ (when x is negative)



Sketching Functions of two variables

1. State the domain of z
2. Sketch several traces on z
3. Sketch the xz and yz traces

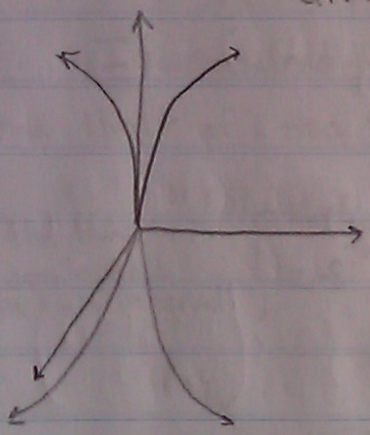
(36) $z = \frac{1}{2}\sqrt{x^2+y^2}$

1. $x^2+y^2 \geq 0$

   all $(x,y) \in (-\infty, \infty)$

2. $1 = \frac{1}{2}\sqrt{x^2+y^2} \Rightarrow 2 = \sqrt{x^2+y^2} \Rightarrow 4 = x^2+y^2$ (circle $r=2$)

   $2 = \frac{1}{2}\sqrt{x^2+y^2} \Rightarrow 4 = \sqrt{x^2+y^2} \Rightarrow 16 = x^2+y^2$ (circle $r=4$)

Same behavior in negative direction,

circles of increasing radius