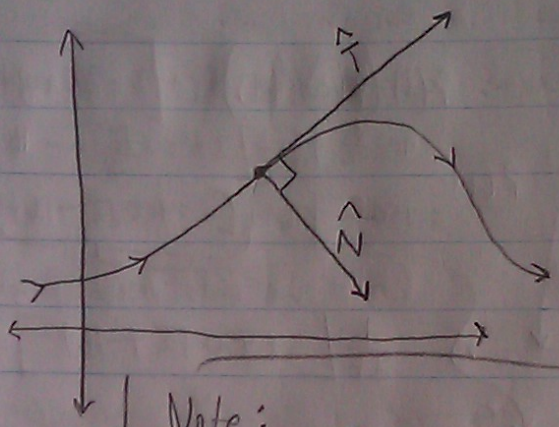
12.4 Tangent and Normal Vectors

Unit tangent vector: (remember: unit vector)

$$\hat{T}(t) = \frac{\hat{r}'(t)}{\|\hat{r}'(t)\|} \qquad \hat{r}'(t) \neq \hat{0}$$

Unit normal vector:

$$\hat{N}(t) = \frac{\hat{T}'(t)}{\|\hat{T}'(t)\|} \qquad \hat{T}'(t) \neq \hat{0}$$



Note:
No concavity $\Rightarrow$ no normal vector
(a line cannot have a normal vector)

Example Let $\hat{r}(t) = (2-t)\hat{i} + \sqrt{t}\,\hat{j}$

Find the unit tangent and the unit normal vectors

$$\hat{r}'(t) = -1\hat{i} + \frac{1}{2\sqrt{t}}\hat{j}$$

$$\|\hat{r}'(t)\| = \sqrt{(-1)^2 + \left(\frac{1}{2\sqrt{t}}\right)^2} = \sqrt{1 + \frac{1}{4t}}$$

$$\hat{T} = \frac{\hat{r}'(t)}{\|\hat{r}'(t)\|} = \frac{1}{\sqrt{1+\frac{1}{4t}}}\left(-\hat{i} + \frac{1}{2\sqrt{t}}\hat{j}\right) = \frac{-1}{\sqrt{1+\frac{1}{4t}}}\hat{i} + \frac{1}{2\sqrt{t+\frac{1}{4}}}\hat{j}$$

$$= -\left(1+\frac{1}{4t}\right)^{-1/2}\hat{i} + \frac{1}{2}\left(t+\frac{1}{4}\right)^{-1/2}\hat{j}$$

$$\hat{T}'(t) = \frac{1}{2}\left(1+\frac{1}{4}t\right)^{-3/2}\left(\frac{1}{4}\right)\hat{i} - \frac{1}{4}\left(t+\frac{1}{4}\right)^{-3/2}(1)\,\hat{j}$$

$$= \frac{1}{8\left(1+\frac{1}{4}t\right)^{3/2}}\hat{i} - \frac{1}{4\left(t+\frac{1}{4}\right)^{3/2}}\hat{j}$$

continued

$$\|\hat{T}'(t)\| = \sqrt{\frac{1}{64(1+\frac{1}{4}t)^3} + \frac{1}{16(t+\frac{1}{4})^3}}$$

Plug in 2

$$\hat{N}(t) = \frac{\hat{T}'(t)}{\|\hat{T}'(t)\|}$$

HW: Concentrate on Polynomial and Trig. forms

Notes: positive slope, increasing acceleration (↙)
negative slope, decreasing acceleration (↘)