12.3 Velocity and Acceleration

If x and y are twice differentiable functions of t then: Position: $\hat{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath}$

Velocity: $\hat{v}(t) = \hat{r}'(t) = x'(t)\hat{\imath} + y'(t)\hat{\jmath}$

Acceleration: $\hat{a}(t) = \hat{r}''(t) = x''(t)\hat{\imath} + y'(t)\hat{\jmath}$

Speed: $\|\hat{v}(t)\| = \sqrt{[x'(t)]^2 + [y'(t)]^2}$

Example Let $\hat{r}(t) = 6 + \hat{\imath} - 4t^2\hat{\jmath} + \sin t\,\hat{k}$

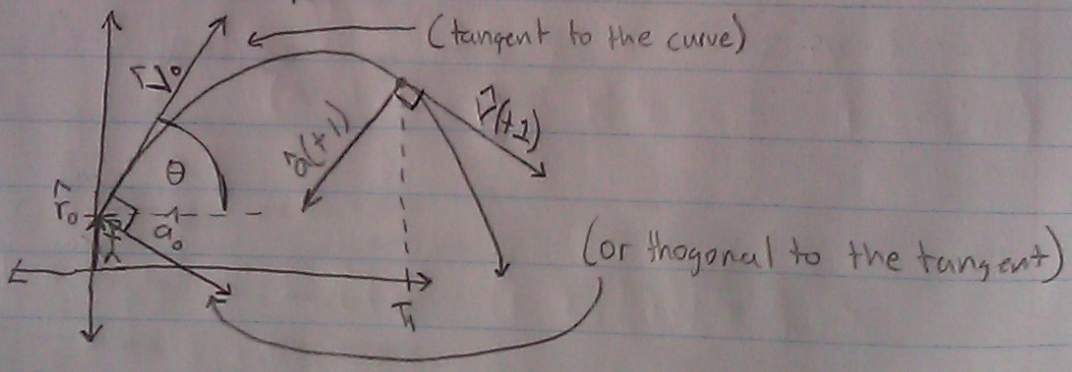
Find $\hat{v}$, $\hat{a}$ and $\|\hat{v}\|$

$\hat{v}(t) = \hat{r}'(t) = 6\hat{\imath} - 8t\hat{\jmath} + \cos t\,\hat{k}$

$\hat{a}(t) = \hat{r}''(t) = 0\hat{\imath} - 8\hat{\jmath} - \sin t\,\hat{k}$

$\|\hat{v}\| = \sqrt{6^2 + (-8t)^2 + (\cos t)^2} = \sqrt{36 + 64t^2 + \cos^2 t}$

General Depiction



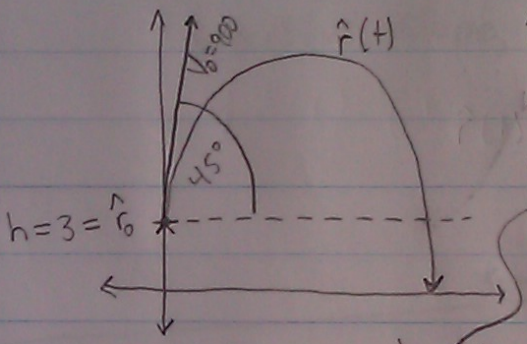
Projectile Motion

$$\hat{r}(t) = -\frac{1}{2} g t^2 \hat{j} + t\hat{V}_0 + \hat{r}_0$$ (position at time t)

or

$$\hat{r}(t) = (V_0 \cos\theta)\, t\, \hat{i} + \left[h + (V_0 \sin\theta)\, t - \tfrac{1}{2} g t^2\right] \hat{j}$$

Example (26) determine the max. height of a projectile fired at a height of 3 ft above the ground at an initial velocity of 900 ft/s at an angle of 45° from the horizontal



$$\hat{r}(t) = (900\cos(45°))\, t\, \hat{i} + \left[3 + 900\sin(45°)\, t - \tfrac{1}{2}(32) t^2\right]$$

$$= 900\tfrac{\sqrt{2}}{2}\, t\, \hat{i} + \left[3 + 900\tfrac{\sqrt{2}}{2}\, t - 16t^2\right] \hat{j}$$

$$= 450\sqrt{2}\, t\, \hat{i} + \left[3 + 450\sqrt{2}\, t - 16t^2\right] \hat{j}$$

(but we only need height ($\hat{j}$))

$$\frac{d}{dt}\left[3 + 450\sqrt{2}\, t - 16t^2\right] = 450\sqrt{2} - 32t$$

$$0 = 450\sqrt{2} - 32t$$

$$t = \frac{450\sqrt{2}}{32}$$

$$t = \frac{225\sqrt{2}}{16} \text{ sec}$$

(into the $\hat{j}$ component)

$$3 + 450\sqrt{2}\left(\tfrac{225\sqrt{2}}{16}\right) - 16t^2$$

$$= 6331.13 \text{ ft} \quad \text{maximum height}$$