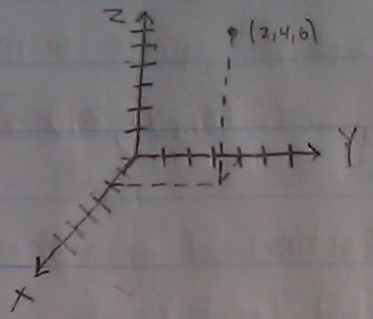
7/7/2009 11.2

11.2) Space Coordinates and Vectors in Space ($\mathbb{R}^3$)



Plot (2, 4, 6)

Space or the three dimensional system of reals ($\mathbb{R}^3$) holds coordinate triples (x, y, z)

when $z=0$, xy-plane

when $y=0$, xz-plane

when $x=0$, yz-plane

Distance Between Two Points

Let $P(p_1, p_2, p_3)$ and $Q(q_1, q_2, q_3)$ be points in $\mathbb{R}^3$

$$d = \sqrt{(q_1-p_1)^2 + (q_2-p_2)^2 + (q_3-p_3)^2}$$

Example

$P(2,-4,3)$ $Q=(-1,0,6)$

$$d = \sqrt{(-1-2)^2 + (0--4)^2 + (6-3)^2}$$
$$= \sqrt{(-3)^2 + (4)^2 + (3)^2}$$
$$= \sqrt{9+16+9}$$
$$= \sqrt{34}$$

The Standard Form of the Eq. of Sphere

$r^2 = (x-h)^2 + (y-k)^2$ circle with center (h,k) and radius r

$r^2 = (x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2$ sphere with center (x_1, y_1, z_1) radius r

Example: write the equation of a sphere with a center $(-1,4,0)$ and radius 6

$(6)^2 = (x-(-1))^2 + (y-(4))^2 + (z-(0))^2$

$\boxed{36 = (x+1)^2 + (y-4)^2 + z^2}$

Example: Find the center and radius of the sphere defined by the equation:

$(x-2)^2 + (y+6)^2 + (z-1)^2 = 49$

$\boxed{\text{center: } (2,-6,1) \quad r = 7}$

Example: Complete the square to find the standard form of the equation for a sphere: $x^2+y^2+z^2+9x-2y+10z+19=0$

Group: $x^2+9x + y^2-2y + z^2+10z = -19$

$\left(\frac{1}{2}(9)\right)^2 = \left(\frac{9}{2}\right)^2 = \left(\frac{81}{4}\right)$ $\quad \left(\frac{1}{2}(-2)\right)^2 = (-1)^2 = 1$ $\quad \left(\frac{1}{2}(10)\right)^2 = (5)^2 = 25$

$\left(x^2+9x+\frac{81}{4}\right) + (y^2-2y+1) + (z^2+10z+25) = -19 + \frac{81}{4} + 1 + 25$

$\left(x+\frac{9}{2}\right)\left(x+\frac{9}{2}\right) + (y-1)(y-1) + (z+5)(z+5) = \frac{-76}{4} + \frac{81}{4} + \frac{4}{4} + \frac{100}{4}$

$\left(x+\frac{9}{2}\right)^2 + (y-1)^2 + (z+5)^2 = \frac{109}{4}$

$\boxed{\text{center: } \left(-\frac{9}{2}, 1, -5\right)} \quad r = \sqrt{\frac{109}{4}} = \boxed{\frac{\sqrt{109}}{2}}$

Midpoint

Take the average of x values, then y values, then z values.

Example: Describe the following:

① $x^2+y^2+z^2 < 9$

inside sphere of radius 3 and center at origin

② $x^2+y^2+z^2 \geq 9$

the sphere and surrounding universe

Vectors in Space

Let $\vec{u} = \langle u_1, u_2, u_3 \rangle$ and $\vec{v} = \langle v_1, v_2, v_3 \rangle$ be vectors in $\mathbb{R}^3$ and let c be scalar

1) $\vec{u} = \vec{v}$ if $u_1 = v_1$, $u_2 = v_2$, $u_3 = v_3$

2) P initial Q terminal

$$\vec{v} = \langle q_1 - p_1, q_2 - p_2, q_3 - p_3 \rangle$$

3) Magnitude: $\|\vec{v}\| = \sqrt{v_1^2 + v_2^2 + v_3^2}$

4) Unit vector in the direction of $\vec{v}$:

$$\frac{1}{\|\vec{v}\|} \cdot \vec{v}$$

5) Vector addition:

$$\vec{u} + \vec{v} = \langle u_1 + v_1, u_2 + v_2, u_3 + v_3 \rangle$$

Scalar Multiplication:

$$c\vec{u} = \langle cu_1, cu_2, cu_3 \rangle$$

6) Standard unit vector notation:

$\hat{i} + \hat{j} + \hat{k}$

$\hat{i} = \langle 1,0,0 \rangle$

$\hat{j} = \langle 0,1,0 \rangle$

$\hat{k} = \langle 0,0,1 \rangle$

Example (assume initial point) (assume terminal point)

$(4,2,-1)$ $(-2,1,4)$

$\vec{v} = \langle -2-4, 1-2, 4+1 \rangle$

$= \langle -6, -1, 5 \rangle$

Magnitude?

$$\|\vec{v}\| = \sqrt{(-6)^2 + (-1)^2 + (5)^2}$$

$$= \sqrt{36 + 1 + 25}$$

$$= \sqrt{62}$$

Unit Vector in the direction of $\vec{v}$

$$\vec{u} = \frac{1}{\|\vec{v}\|}\vec{v} = \frac{1}{\sqrt{62}}\langle -6, -1, 5 \rangle$$

$$= \left\langle \frac{-6}{\sqrt{62}}, \frac{-1}{\sqrt{62}}, \frac{5}{\sqrt{62}} \right\rangle$$

$$= \left\langle \frac{-6\sqrt{62}}{62}, \frac{-1\sqrt{62}}{62}, \frac{5\sqrt{62}}{62} \right\rangle$$

$$= \left\langle \frac{-3\sqrt{62}}{31}, \frac{-1\sqrt{62}}{62}, \frac{5\sqrt{62}}{62} \right\rangle$$

Parallel Vectors

Two nonzero vectors $\vec{u}$ and $\vec{v}$ are parallel if there exists a c such that either $\vec{u} = c\vec{v}$ or $\vec{v} = c\vec{u}$

Example: Determine if the following are parallel:

① $6\hat{i} - 4\hat{j} + 9\hat{k}$

$\frac{1}{2}\hat{i} - \frac{2}{3} + \frac{3}{4}\hat{k} \rightarrow 12\left(\frac{1}{2}\hat{i} - \frac{2}{3}\hat{j} + \frac{3}{4}\hat{k}\right)$

$(6\hat{i} - 8\hat{j}$

not 4 so...

Not parallel

② $-\hat{i} + \frac{4}{3}\hat{j} - \frac{3}{2}\hat{k}$

$\frac{1}{2}\hat{i} - \frac{2}{3}\hat{j} + \frac{3}{4}\hat{k} \rightarrow -2\left(\frac{1}{2}\hat{i} - \frac{2}{3}\hat{j} + \frac{3}{4}\hat{k}\right)$

$= -\hat{i} + \frac{4}{3}\hat{j} - \frac{3}{2}\hat{k}$ ✓

Parallel mult. $c = -2$