CHAPTER 11
Vectors and the Geometry of Space

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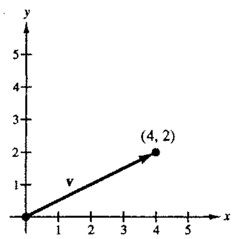
Section 11.1 Vectors in the Plane

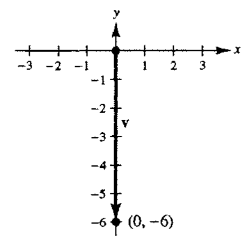
1. (a) $\mathbf{v} = \langle 5 - 1, 3 - 1 \rangle = \langle 4, 2 \rangle$

(b)



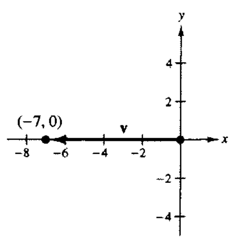
2. (a) $\mathbf{v} = \langle 3 - 3, -2 - 4 \rangle = \langle 0, -6 \rangle$

(b)



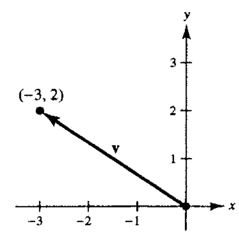
3. (a) $\mathbf{v} = \langle -4 - 3, -2 - (-2) \rangle = \langle -7, 0 \rangle$

(b)



4. (a) $\mathbf{v} = \langle -1 - 2, 3 - 1 \rangle = \langle -3, 2 \rangle$

(b)



5. $\mathbf{u} = \langle 5 - 3, 6 - 2 \rangle = \langle 2, 4 \rangle$

$\mathbf{v} = \langle 1 - (-1), 8 - 4 \rangle = \langle 2, 4 \rangle$

$\mathbf{u} = \mathbf{v}$

6. $\mathbf{u} = \langle 1 - (-4), 8 - 0 \rangle = \langle 5, 8 \rangle$

$\mathbf{v} = \langle 7 - 2, 7 - (-1) \rangle = \langle 5, 8 \rangle$

$\mathbf{u} = \mathbf{v}$

7. $\mathbf{u} = \langle 6 - 0, -2 - 3 \rangle = \langle 6, -5 \rangle$

$\mathbf{v} = \langle 9 - 3, 5 - 10 \rangle = \langle 6, -5 \rangle$

$\mathbf{u} = \mathbf{v}$

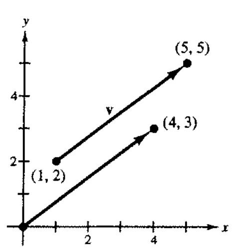
8. $\mathbf{u} = \langle 11 - (-4), -4 - (-1) \rangle = \langle 15, -3 \rangle$

$\mathbf{v} = \langle 25 - 0, 10 - 13 \rangle = \langle 15, -3 \rangle$

$\mathbf{u} = \mathbf{v}$

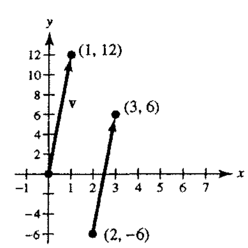
9. (b) $\mathbf{v} = \langle 5 - 1, 5 - 2 \rangle = \langle 4, 3 \rangle$

(a) and (c).



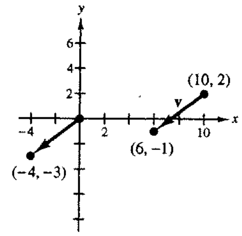
10. (b) $\mathbf{v} = \langle 3 - 2, 6 - (-6) \rangle = \langle 1, 12 \rangle$

(a) and (c).



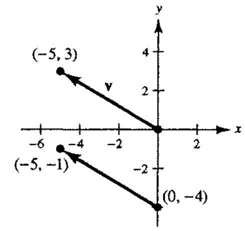
11. (b) $\mathbf{v} = \langle 6 - 10, -1 - 2 \rangle = \langle -4, -3 \rangle$

(a) and (c).



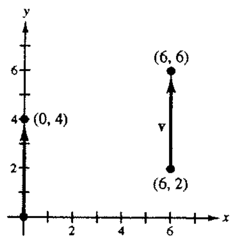
12. (b) $\mathbf{v} = \langle -5 - 0, -1 - (-4) \rangle = \langle -5, 3 \rangle$

(a) and (c).



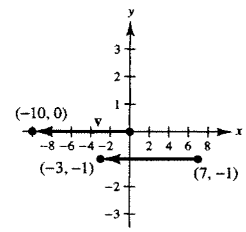
13. (b) $\mathbf{v} = \langle 6 - 6, 6 - 2 \rangle = \langle 0, 4 \rangle$

(a) and (c).



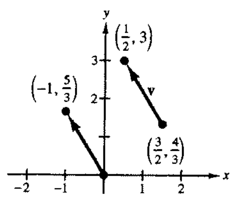
14. (b) $\mathbf{v} = \langle -3 - 7, -1 - (-1) \rangle = \langle -10, 0 \rangle$

(a) and (c).



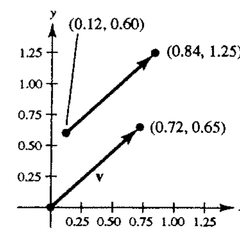
15. (b) $\mathbf{v} = \left\langle \frac{1}{2} - \frac{3}{2}, 3 - \frac{4}{3} \right\rangle = \left\langle -1, \frac{5}{3} \right\rangle$

(a) and (c).

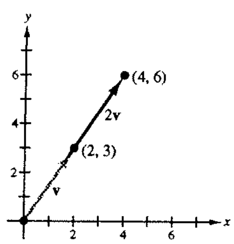


16. (b) $\mathbf{v} = \langle 0.84 - 0.12, 1.25 - 0.60 \rangle = \langle 0.72, 0.65 \rangle$

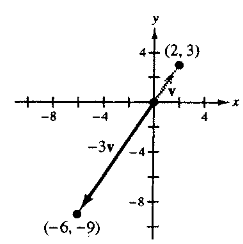
(a) and (c).



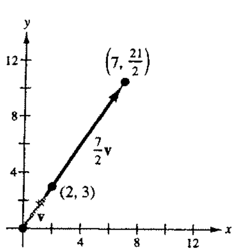
17. (a) $2\mathbf{v} = \langle 4, 6 \rangle$



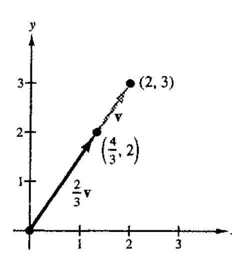
(b) $-3\mathbf{v} = \langle -6, -9 \rangle$



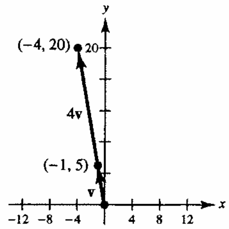
(c) $\frac{7}{2}\mathbf{v} = \left\langle 7, \frac{21}{2} \right\rangle$



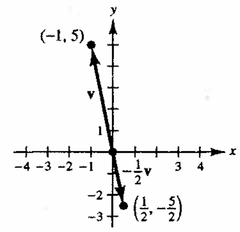
(d) $\frac{2}{3}\mathbf{v} = \left\langle \frac{4}{3}, 2 \right\rangle$



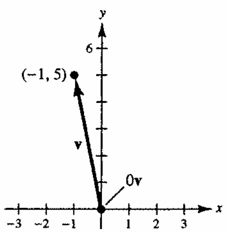
18. (a) $4\mathbf{v} = \langle -4, 20\rangle$



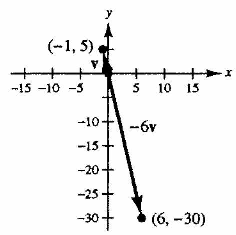
(b) $-\frac{1}{2}\mathbf{v} = \left\langle \frac{1}{2}, -\frac{5}{2}\right\rangle$



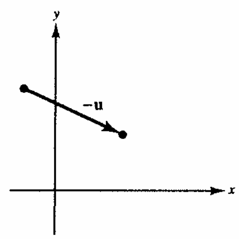
(c) $0\mathbf{v} = \langle 0, 0\rangle$



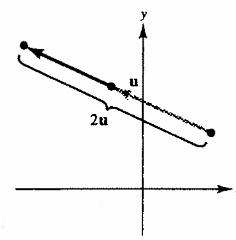
(d) $-6\mathbf{v} = \langle 6, -30\rangle$



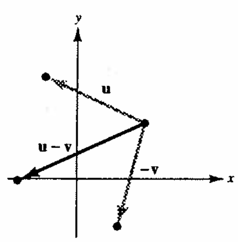
19.



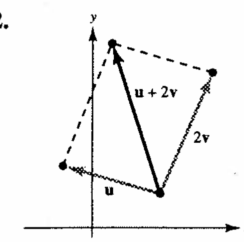
20. Twice as long as given vector **u**.



21.



22.



23. (a) $\frac{2}{3}\mathbf{u} = \frac{2}{3}\langle 4, 9\rangle = \left\langle \frac{8}{3}, 6\right\rangle$

(b) $\mathbf{v} - \mathbf{u} = \langle 2, -5\rangle - \langle 4, 9\rangle = \langle -2, -14\rangle$

(c) $2\mathbf{u} + 5\mathbf{v} = 2\langle 4, 9\rangle + 5\langle 2, -5\rangle = \langle 18, -7\rangle$

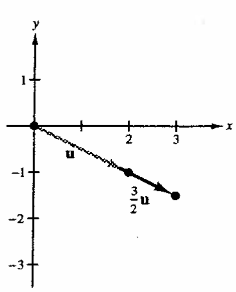
24. (a) $\frac{2}{3}\mathbf{u} = \frac{2}{3}\langle -3, -8\rangle = \left\langle -2, -\frac{16}{3}\right\rangle$

(b) $\mathbf{v} - \mathbf{u} = \langle 8, 25\rangle - \langle -3, -8\rangle = \langle 11, 33\rangle$

(c) $2\mathbf{u} + 5\mathbf{v} = 2\langle -3, -8\rangle + 5\langle 8, 25\rangle = \langle 34, 109\rangle$

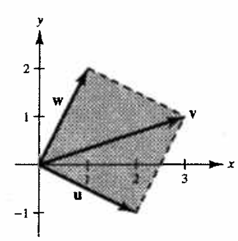
25. $\mathbf{v} = \frac{3}{2}(2\mathbf{i} - \mathbf{j}) = 3\mathbf{i} - \frac{3}{2}\mathbf{j}$

$= \left\langle 3, -\frac{3}{2}\right\rangle$



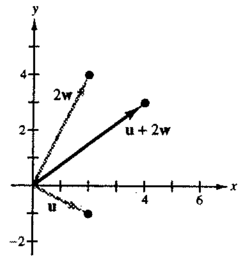
26. $\mathbf{v} = (2\mathbf{i} - \mathbf{j}) + (\mathbf{i} + 2\mathbf{j})$

$= 3\mathbf{i} + \mathbf{j} = \langle 3, 1\rangle$



27. $\mathbf{v} = (2\mathbf{i} - \mathbf{j}) + 2(\mathbf{i} + 2\mathbf{j})$

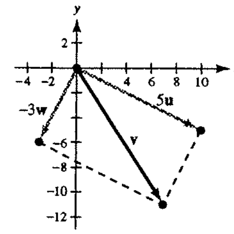
$= 4\mathbf{i} + 3\mathbf{j} = \langle 4, 3\rangle$



28. $\mathbf{v} = 5\mathbf{u} - 3\mathbf{w}$

$= 5\langle 2, -1\rangle - 3\langle 1, 2\rangle$

$= \langle 7, -11\rangle$



29. $u_1 - 4 = -1 \qquad u_1 = 3$

$u_2 - 2 = 3 \qquad u_2 = 5$

$Q = (3, 5)$

30. $u_1 - 3 = 4 \qquad u_1 = 7$

$u_2 - 2 = -9 \qquad u_2 = -7$

$Q = (7, -7)$

31. $\|\mathbf{v}\| = \sqrt{16 + 9} = 5$

32. $\|\mathbf{v}\| = \sqrt{144 + 25} = 13$

33. $\|\mathbf{v}\| = \sqrt{36 + 25} = \sqrt{61}$

34. $\|\mathbf{v}\| = \sqrt{100 + 9} = \sqrt{109}$

35. $\|\mathbf{v}\| = \sqrt{0 + 16} = 4$

36. $\|\mathbf{v}\| = \sqrt{1 + 1} = \sqrt{2}$

37. $\|\mathbf{u}\| = \sqrt{3^2 + 12^2} = \sqrt{153}$

$$\mathbf{v} = \frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{\langle 3, 12\rangle}{\sqrt{153}} = \left\langle \frac{3}{\sqrt{153}}, \frac{12}{\sqrt{153}} \right\rangle$$

$$= \left\langle \frac{\sqrt{17}}{17}, \frac{4\sqrt{17}}{17} \right\rangle \text{ unit vector}$$

38. $\|\mathbf{u}\| = \sqrt{5^2 + 15^2} = \sqrt{250} = 5\sqrt{10}$

$$\mathbf{v} = \frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{\langle 5, 15\rangle}{5\sqrt{10}} = \left\langle \frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}} \right\rangle \text{ unit vector}$$

39. $\|\mathbf{u}\| = \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{5}{2}\right)^2} = \frac{\sqrt{34}}{2}$

$$\mathbf{v} = \frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{\langle (3/2), (5/2)\rangle}{\sqrt{34}/2} = \left\langle \frac{3}{\sqrt{34}}, \frac{5}{\sqrt{34}} \right\rangle$$

$$= \left\langle \frac{3\sqrt{34}}{34}, \frac{5\sqrt{34}}{34} \right\rangle \text{ unit vector}$$

40. $\|\mathbf{u}\| = \sqrt{(-6.2)^2 + (3.4)^2} = \sqrt{50} = 5\sqrt{2}$

$$\mathbf{v} = \frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{\langle -6.2, 3.4\rangle}{5\sqrt{2}} = \left\langle \frac{-1.24}{\sqrt{2}}, \frac{0.68}{\sqrt{2}} \right\rangle \text{ unit vector}$$

41. $\mathbf{u} = \langle 1, -1\rangle, \mathbf{v} = \langle -1, 2\rangle$

(a) $\|\mathbf{u}\| = \sqrt{1 + 1} = \sqrt{2}$

(b) $\|\mathbf{v}\| = \sqrt{1 + 4} = \sqrt{5}$

(c) $\mathbf{u} + \mathbf{v} = \langle 0, 1\rangle$

$\|\mathbf{u} + \mathbf{v}\| = \sqrt{0 + 1} = 1$

(d) $\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = \dfrac{1}{\sqrt{2}}\langle 1, -1\rangle$

$\left\|\dfrac{\mathbf{u}}{\|\mathbf{u}\|}\right\| = 1$

(e) $\dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \dfrac{1}{\sqrt{5}}\langle -1, 2\rangle$

$\left\|\dfrac{\mathbf{v}}{\|\mathbf{v}\|}\right\| = 1$

(f) $\dfrac{\mathbf{u} + \mathbf{v}}{\|\mathbf{u} + \mathbf{v}\|} = \langle 0, 1\rangle$

$\left\|\dfrac{\mathbf{u} + \mathbf{v}}{\|\mathbf{u} + \mathbf{v}\|}\right\| = 1$

42. $\mathbf{u} = \langle 0, 1\rangle, \mathbf{v} = \langle 3, -3\rangle$

(a) $\|\mathbf{u}\| = \sqrt{0 + 1} = 1$

(b) $\|\mathbf{v}\| = \sqrt{9 + 9} = 3\sqrt{2}$

(c) $\mathbf{u} + \mathbf{v} = \langle 3, -2\rangle$

$\|\mathbf{u} + \mathbf{v}\| = \sqrt{9 + 4} = \sqrt{13}$

(d) $\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = \langle 0, 1\rangle$

$\left\|\dfrac{\mathbf{u}}{\|\mathbf{u}\|}\right\| = 1$

(e) $\dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \dfrac{1}{3\sqrt{2}}\langle 3, -3\rangle$

$\left\|\dfrac{\mathbf{v}}{\|\mathbf{v}\|}\right\| = 1$

(f) $\dfrac{\mathbf{u} + \mathbf{v}}{\|\mathbf{u} + \mathbf{v}\|} = \dfrac{1}{\sqrt{13}}\langle 3, -2\rangle$

$\left\|\dfrac{\mathbf{u} + \mathbf{v}}{\|\mathbf{u} + \mathbf{v}\|}\right\| = 1$

43. $\mathbf{u} = \left\langle 1, \frac{1}{2} \right\rangle, \mathbf{v} = \langle 2, 3 \rangle$

(a) $\|\mathbf{u}\| = \sqrt{1 + \frac{1}{4}} = \frac{\sqrt{5}}{2}$

(b) $\|\mathbf{v}\| = \sqrt{4 + 9} = \sqrt{13}$

(c) $\mathbf{u} + \mathbf{v} = \left\langle 3, \frac{7}{2} \right\rangle$

$\|\mathbf{u} + \mathbf{v}\| = \sqrt{9 + \frac{49}{4}} = \frac{\sqrt{85}}{2}$

(d) $\frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{2}{\sqrt{5}} \left\langle 1, \frac{1}{2} \right\rangle$

$\left\| \frac{\mathbf{u}}{\|\mathbf{u}\|} \right\| = 1$

(e) $\frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{1}{\sqrt{13}} \langle 2, 3 \rangle$

$\left\| \frac{\mathbf{v}}{\|\mathbf{v}\|} \right\| = 1$

(f) $\frac{\mathbf{u} + \mathbf{v}}{\|\mathbf{u} + \mathbf{v}\|} = \frac{2}{\sqrt{85}} \left\langle 3, \frac{7}{2} \right\rangle$

$\left\| \frac{\mathbf{u} + \mathbf{v}}{\|\mathbf{u} + \mathbf{v}\|} \right\| = 1$

44. $\mathbf{u} = \langle 2, -4 \rangle, \mathbf{v} = \langle 5, 5 \rangle$

(a) $\|\mathbf{u}\| = \sqrt{4 + 16} = 2\sqrt{5}$

(b) $\|\mathbf{v}\| = \sqrt{25 + 25} = 5\sqrt{2}$

(c) $\mathbf{u} + \mathbf{v} = \langle 7, 1 \rangle$

$\|\mathbf{u} + \mathbf{v}\| = \sqrt{49 + 1} = 5\sqrt{2}$

(d) $\frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{1}{2\sqrt{5}} \langle 2, -4 \rangle$

$\left\| \frac{\mathbf{u}}{\|\mathbf{u}\|} \right\| = 1$

(e) $\frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{1}{5\sqrt{2}} \langle 5, 5 \rangle$

$\left\| \frac{\mathbf{v}}{\|\mathbf{v}\|} \right\| = 1$

(f) $\frac{\mathbf{u} + \mathbf{v}}{\|\mathbf{u} + \mathbf{v}\|} = \frac{1}{5\sqrt{2}} \langle 7, 1 \rangle$

$\left\| \frac{\mathbf{u} + \mathbf{v}}{\|\mathbf{u} + \mathbf{v}\|} \right\| = 1$

45. $\mathbf{u} = \langle 2, 1 \rangle$

$\|\mathbf{u}\| = \sqrt{5} \approx 2.236$

$\mathbf{v} = \langle 5, 4 \rangle$

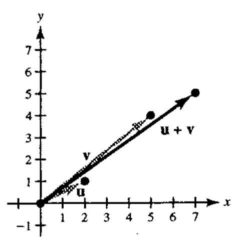
$\|\mathbf{v}\| = \sqrt{41} \approx 6.403$

$\mathbf{u} + \mathbf{v} = \langle 7, 5 \rangle$

$\|\mathbf{u} + \mathbf{v}\| = \sqrt{74} \approx 8.602$

$\|\mathbf{u} + \mathbf{v}\| \le \|\mathbf{u}\| + \|\mathbf{v}\|$

$\sqrt{74} \le \sqrt{5} + \sqrt{41}$



46. $\mathbf{u} = \langle -3, 2 \rangle$

$\|\mathbf{u}\| = \sqrt{13} \approx 3.606$

$\mathbf{v} = \langle 1, -2 \rangle$

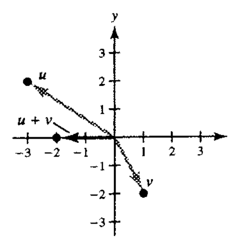
$\|\mathbf{v}\| = \sqrt{5} \approx 2.236$

$\mathbf{u} + \mathbf{v} = \langle -2, 0 \rangle$

$\|\mathbf{u} + \mathbf{v}\| = 2$

$\|\mathbf{u} + \mathbf{v}\| \le \|\mathbf{u}\| + \|\mathbf{v}\|$

$2 \le \sqrt{13} + \sqrt{5}$



47. $\frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{1}{\sqrt{2}} \langle 1, 1 \rangle$

$4\left(\frac{\mathbf{u}}{\|\mathbf{u}\|}\right) = 2\sqrt{2} \langle 1, 1 \rangle$

$\mathbf{v} = \left\langle 2\sqrt{2}, 2\sqrt{2} \right\rangle$

48. $\frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{1}{\sqrt{2}} \langle -1, 1 \rangle$

$4\left(\frac{\mathbf{u}}{\|\mathbf{u}\|}\right) = 2\sqrt{2} \langle -1, 1 \rangle$

$\mathbf{v} = \left\langle -2\sqrt{2}, 2\sqrt{2} \right\rangle$

49. $\frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{1}{2\sqrt{3}} \left\langle \sqrt{3}, 3 \right\rangle$

$2\left(\frac{\mathbf{u}}{\|\mathbf{u}\|}\right) = \frac{1}{\sqrt{3}} \left\langle \sqrt{3}, 3 \right\rangle$

$\mathbf{v} = \left\langle 1, \sqrt{3} \right\rangle$

50. $\frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{1}{3} \langle 0, 3 \rangle$

$3\left(\frac{\mathbf{u}}{\|\mathbf{u}\|}\right) = \langle 0, 3 \rangle$

$\mathbf{v} = \langle 0, 3 \rangle$

51. $\mathbf{v} = 3[(\cos 0°)\mathbf{i} + (\sin 0°)\mathbf{j}]$

$= 3\mathbf{i} = \langle 3, 0 \rangle$

52. $\mathbf{v} = 5[(\cos 120°)\mathbf{i} + (\sin 120°)\mathbf{j}]$

$= -\frac{5}{2}\mathbf{i} + \frac{5\sqrt{3}}{2}\mathbf{j}$

53. $\mathbf{v} = 2[(\cos 150°)\mathbf{i} + (\sin 150°)\mathbf{j}]$

$= -\sqrt{3}\mathbf{i} + \mathbf{j} = \left\langle -\sqrt{3}, 1 \right\rangle$

54. $\mathbf{v} = (\cos 3.5°)\mathbf{i} + (\sin 3.5°)\mathbf{j}$

$\approx 0.9981\mathbf{i} + 0.0610\mathbf{j} = \langle 0.9981, 0.0610 \rangle$

55.
$$\mathbf{u} = \mathbf{i}$$
$$\mathbf{v} = \frac{3\sqrt{2}}{2}\mathbf{i} + \frac{3\sqrt{2}}{2}\mathbf{j}$$
$$\mathbf{u} + \mathbf{v} = \left(\frac{2 + 3\sqrt{2}}{2}\right)\mathbf{i} + \frac{3\sqrt{2}}{2}\mathbf{j}$$

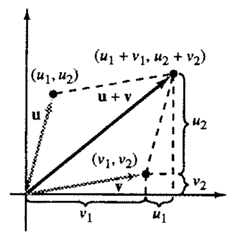
56.
$$\mathbf{u} = 4\mathbf{i}$$
$$\mathbf{v} = \mathbf{i} + \sqrt{3}\mathbf{j}$$
$$\mathbf{u} + \mathbf{v} = 5\mathbf{i} + \sqrt{3}\mathbf{j}$$

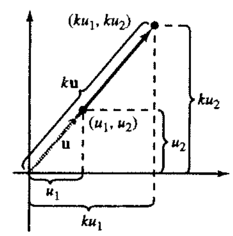
57.
$$\mathbf{u} = 2(\cos 4)\mathbf{i} + 2(\sin 4)\mathbf{j}$$
$$\mathbf{v} = (\cos 2)\mathbf{i} + (\sin 2)\mathbf{j}$$
$$\mathbf{u} + \mathbf{v} = (2\cos 4 + \cos 2)\mathbf{i} + (2\sin 4 + \sin 2)\mathbf{j}$$

58.
$$\mathbf{u} = 5[\cos(-0.5)]\mathbf{i} + 5[\sin(-0.5)]\mathbf{j}$$
$$= 5[\cos(0.5)]\mathbf{i} - 5[\sin(0.5)]\mathbf{j}$$
$$\mathbf{v} = 5[\cos(0.5)]\mathbf{i} + 5[\sin(0.5)]\mathbf{j}$$
$$\mathbf{u} + \mathbf{v} = 10[\cos(0.5)]\mathbf{i}$$

59. A scalar is a real number. A vector is represented by a directed line segment. A vector has both length and direction.

60. See page 764:





61. (a) Vector. The velocity has both magnitude and direction.

(b) Scalar. The price is a number.

62. (a) Scalar. The temperature is a number.

(b) Vector. The weight has magnitude and direction.

For Exercises 63–68, $a\mathbf{u} + b\mathbf{w} = a(\mathbf{i} + 2\mathbf{j}) + b(\mathbf{i} - \mathbf{j}) = (a + b)\mathbf{i} + (2a - b)\mathbf{j}$.

63. $\mathbf{v} = 2\mathbf{i} + \mathbf{j}$. Therefore, $a + b = 2$, $2a - b = 1$. Solving simultaneously, we have $a = 1$, $b = 1$.

64. $\mathbf{v} = 3\mathbf{j}$. Therefore, $a + b = 0$, $2a - b = 3$. Solving simultaneously, we have $a = 1$, $b = -1$.

65. $\mathbf{v} = 3\mathbf{i}$. Therefore, $a + b = 3$, $2a - b = 0$. Solving simultaneously, we have $a = 1$, $b = 2$.

66. $\mathbf{v} = 3\mathbf{i} + 3\mathbf{j}$. Therefore, $a + b = 3$, $2a - b = 3$. Solving simultaneously, we have $a = 2$, $b = 1$.

67. $\mathbf{v} = \mathbf{i} + \mathbf{j}$. Therefore, $a + b = 1$, $2a - b = 1$. Solving simultaneously, we have $a = \frac{2}{3}$, $b = \frac{1}{3}$.

68. $\mathbf{v} = -\mathbf{i} + 7\mathbf{j}$. Therefore, $a + b = -1$, $2a - b = 7$. Solving simultaneously, we have $a = 2$, $b = -3$.

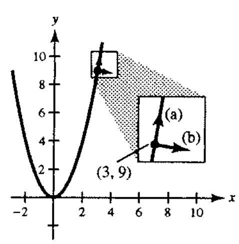
69. $f(x) = x^2$, $f'(x) = 2x$, $f'(3) = 6$

(a) $m = 6$. Let $\mathbf{w} = \langle 1, 6\rangle$, $\|\mathbf{w}\| = \sqrt{37}$.

$$\pm\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{\sqrt{37}}\langle 1, 6\rangle \text{ unit tangent vectors}$$

(b) $m = -\frac{1}{6}$. Let $\mathbf{w} = \langle -6, 1\rangle$, $\|\mathbf{w}\| = \sqrt{37}$.

$$\pm\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{\sqrt{37}}\langle -6, 1\rangle \text{ unit normal vectors}$$



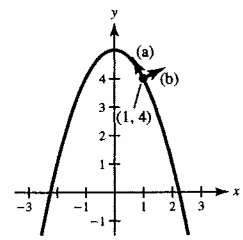
70. $f(x) = -x^2 + 5, f'(x) = -2x, f'(1) = -2$

(a) $m = -2$. Let $\mathbf{w} = \langle 1, -2\rangle, \|\mathbf{w}\| = \sqrt{5}$.

$$\pm\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{\sqrt{5}}\langle 1, -2\rangle \text{ unit tangent vectors}$$

(b) $m = \frac{1}{2}$. Let $\mathbf{w} = \langle 2, 1\rangle, \|\mathbf{w}\| = \sqrt{5}$.

$$\pm\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{\sqrt{5}}\langle 2, 1\rangle \text{ unit normal vectors}$$



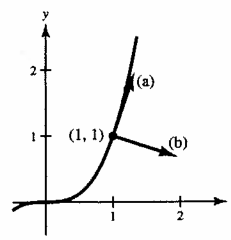
71. $f(x) = x^3, f'(x) = 3x^2 = 3$ at $x = 1$.

(a) $m = 3$. Let $\mathbf{w} = \langle 1, 3\rangle$, then

$$\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{\sqrt{10}}\langle 1, 3\rangle.$$

(b) $m = -\frac{1}{3}$. Let $\mathbf{w} = \langle 3, -1\rangle$, then

$$\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{\sqrt{10}}\langle 3, -1\rangle.$$



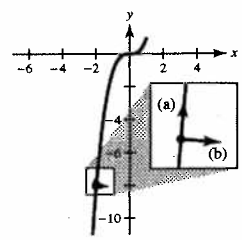
72. $f(x) = x^3, f'(x) = 3x^2 = 12$ at $x = -2$.

(a) $m = 12$. Let $\mathbf{w} = \langle 1, 12\rangle$, then

$$\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{\sqrt{145}}\langle 1, 12\rangle.$$

(b) $m = -\frac{1}{12}$. Let $\mathbf{w} = \langle 12, -1\rangle$, then

$$\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{\sqrt{145}}\langle 12, -1\rangle.$$



73. $f(x) = \sqrt{25 - x^2}$

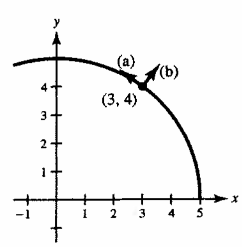
$$f'(x) = \frac{-x}{\sqrt{25 - x^2}} = \frac{-3}{4} \text{ at } x = 3.$$

(a) $m = -\frac{3}{4}$. Let $\mathbf{w} = \langle -4, 3\rangle$, then

$$\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{5}\langle -4, 3\rangle.$$

(b) $m = \frac{4}{3}$. Let $\mathbf{w} = \langle 3, 4\rangle$, then

$$\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{5}\langle 3, 4\rangle.$$



74. $f(x) = \tan x$

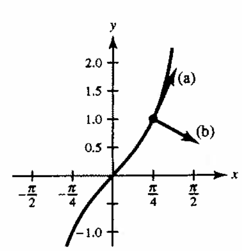
$$f'(x) = \sec^2 x = 2 \text{ at } x = \frac{\pi}{4}.$$

(a) $m = 2$. Let $\mathbf{w} = \langle 1, 2\rangle$, then

$$\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{\sqrt{5}}\langle 1, 2\rangle.$$

(b) $m = -\frac{1}{2}$. Let $\mathbf{w} = \langle -2, 1\rangle$, then

$$\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{\sqrt{5}}\langle -2, 1\rangle.$$



75. $\mathbf{u} = \frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j}$

$\mathbf{u} + \mathbf{v} = \sqrt{2}\mathbf{j}$

$\mathbf{v} = (\mathbf{u} + \mathbf{v}) - \mathbf{u} = -\frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j}$

76. $\mathbf{u} = 2\sqrt{3}\mathbf{i} + 2\mathbf{j}$

$\mathbf{u} + \mathbf{v} = -3\mathbf{i} + 3\sqrt{3}\mathbf{j}$

$\mathbf{v} = (\mathbf{u} + \mathbf{v}) - \mathbf{u} = \left(-3 - 2\sqrt{3}\right)\mathbf{i} + \left(3\sqrt{3} - 2\right)\mathbf{j}$

77. Programs will vary.

78. Magnitude ≈ 63.5

Direction $\approx -8.26°$

79. $\|\mathbf{F}_1\| = 2,\ \theta_{\mathbf{F}_1} = 33°$

$\|\mathbf{F}_2\| = 3,\ \theta_{\mathbf{F}_2} = -125°$

$\|\mathbf{F}_3\| = 2.5,\ \theta_{\mathbf{F}_3} = 110°$

$\|\mathbf{R}\| = \|\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3\| \approx 1.33$

$\theta_{\mathbf{R}} = \theta_{\mathbf{F}_1+\mathbf{F}_2+\mathbf{F}_3} \approx 132.5°$

80. $\|\mathbf{F}_1\| = 2,\ \theta_{\mathbf{F}_1} = -10°$

$\|\mathbf{F}_2\| = 4,\ \theta_{\mathbf{F}_2} = 140°$

$\|\mathbf{F}_3\| = 3,\ \theta_{\mathbf{F}_3} = 200°$

$\|\mathbf{R}\| = \|\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3\| \approx 4.09$

$\theta_{\mathbf{R}} = \theta_{\mathbf{F}_1+\mathbf{F}_2+\mathbf{F}_3} \approx 163.0°$

81. (a) $180(\cos 30°\mathbf{i} + \sin 30°\mathbf{j}) + 275\mathbf{i} \approx 430.88\mathbf{i} + 90\mathbf{j}$

Direction: $\alpha \approx \arctan\left(\frac{90}{430.88}\right) \approx 0.206\ (\approx 11.8°)$

Magnitude: $\sqrt{430.88^2 + 90^2} \approx 440.18$ newtons

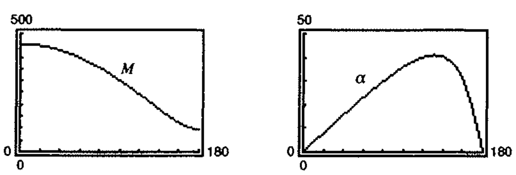
(b) $M = \sqrt{(275 + 180\cos\theta)^2 + (180\sin\theta)^2}$

$\alpha = \arctan\left[\frac{180\sin\theta}{275 + 180\cos\theta}\right]$

(c)

θ	0°	30°	60°	90°	120°	150°	180°
M	455	440.2	396.9	328.7	241.9	149.3	95
α	0°	11.8°	23.1°	33.2°	40.1°	37.1°	0

(d)



(e) M decreases because the forces change from acting in the same direction to acting in the opposite direction as θ increases from 0° to 180°.

82. $\mathbf{F}_1 + \mathbf{F}_2 = (500\cos 30°\mathbf{i} + 500\sin 30°\mathbf{j}) + (200\cos(-45°)\mathbf{i} + 200\sin(-45°)\mathbf{j})$

$= \left(250\sqrt{3} + 100\sqrt{2}\right)\mathbf{i} + \left(250 - 100\sqrt{2}\right)\mathbf{j}$

$\|\mathbf{F}_1 + \mathbf{F}_2\| = \sqrt{\left(250\sqrt{3} + 100\sqrt{2}\right)^2 + \left(250 - 100\sqrt{2}\right)^2} \approx 584.6$ lb

$\tan\theta = \frac{250 - 100\sqrt{2}}{250\sqrt{3} + 100\sqrt{2}} \implies \theta \approx 10.7°$

83. $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = (75\cos 30°\mathbf{i} + 75\sin 30°\mathbf{j}) + (100\cos 45°\mathbf{i} + 100\sin 45°\mathbf{j}) + (125\cos 120°\mathbf{i} + 125\sin 120°\mathbf{j})$

$= \left(\frac{75}{2}\sqrt{3} + 50\sqrt{2} - \frac{125}{2}\right)\mathbf{i} + \left(\frac{75}{2} + 50\sqrt{2} + \frac{125}{2}\sqrt{3}\right)\mathbf{j}$

$\|\mathbf{R}\| = \|\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3\| \approx 228.5$ lb

$\theta_{\mathbf{R}} = \theta_{\mathbf{F}_1+\mathbf{F}_2+\mathbf{F}_3} \approx 71.3°$

84. $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = [400(\cos(-30°)\mathbf{i} + \sin(-30°)\mathbf{j})] + [280(\cos(45°)\mathbf{i} + \sin(45°)\mathbf{j})] + [350(\cos(135°)\mathbf{i} + \sin(135°)\mathbf{j}]$

$$= \left[200\sqrt{3} + 140\sqrt{2} - 175\sqrt{2}\right]\mathbf{i} + \left[-200 + 140\sqrt{2} + 175\sqrt{2}\right]\mathbf{j}$$

$$\|\mathbf{R}\| = \sqrt{(200\sqrt{3} - 35\sqrt{2})^2 + (-200 + 315\sqrt{2})^2} \approx 385.2483 \text{ newtons}$$

$$\theta_{\mathbf{R}} = \arctan\left(\frac{-200 + 315\sqrt{2}}{200\sqrt{3} - 35\sqrt{2}}\right) \approx 0.6908 \approx 39.6°$$

85. (a) The forces act along the same direction. $\theta = 0°$.

(b) The forces cancel out each other. $\theta = 180°$.

(c) No, the magnitude of the resultant can not be greater than the sum.

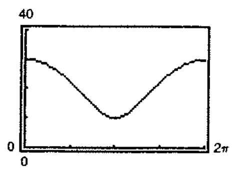
86. $\mathbf{F}_1 = \langle 20, 0\rangle$, $\mathbf{F}_2 = 10\langle \cos\theta, \sin\theta\rangle$

(a) $\|\mathbf{F}_1 + \mathbf{F}_2\| = \|\langle 20 + 10\cos\theta, 10\sin\theta\rangle\|$

$$= \sqrt{400 + 400\cos\theta + 100\cos^2\theta + 100\sin^2\theta}$$

$$= \sqrt{500 + 400\cos\theta}$$

(b)



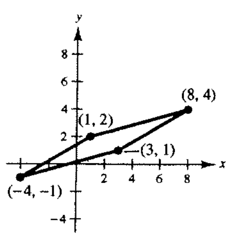
(c) The range is $10 \le \|\mathbf{F}_1 + \mathbf{F}_2\| \le 30$.

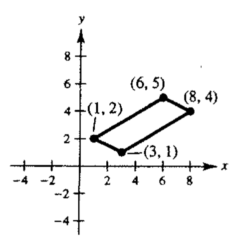
The maximum is 30, which occur at $\theta = 0$ and $\theta = 2\pi$.

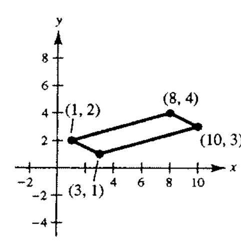
The minimum is 10 at $\theta = \pi$.

(d) The minimum of the resultant is 10.

87. $(-4, -1)$, $(6, 5)$, $(10, 3)$







88. $\mathbf{u} = \langle 7 - 1, 5 - 2\rangle = \langle 6, 3\rangle$

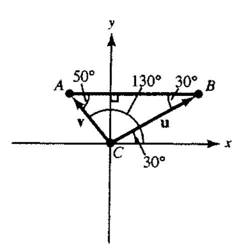
$\frac{1}{3}\mathbf{u} = \langle 2, 1\rangle$

$P_1 = (1, 2) + (2, 1) = (3, 3)$

$P_2 = (1, 2) + 2(2, 1) = (5, 4)$

89. $\mathbf{u} = \overrightarrow{CB} = \|\mathbf{u}\|(\cos 30°\,\mathbf{i} + \sin 30°\,\mathbf{j})$

$\mathbf{v} = \overrightarrow{CA} = \|\mathbf{v}\|(\cos 130°\,\mathbf{i} + \sin 130°\,\mathbf{j})$



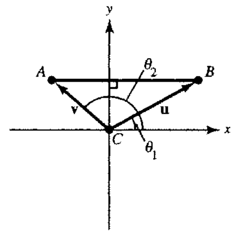
Vertical components: $\|\mathbf{u}\|\sin 30° + \|\mathbf{v}\|\sin 130° = 2000$

Horizontal components: $\|\mathbf{u}\|\cos 30° + \|\mathbf{v}\|\cos 130° = 0$

Solving this system, you obtain

$\|\mathbf{u}\| \approx 1305.5$ pounds and $\|\mathbf{v}\| \approx 1758.8$ pounds.

90. $\theta_1 = \arctan\left(\frac{24}{20}\right) \approx 0.8761$ or $50.2°$



$\theta_2 = \arctan\left(\frac{24}{-10}\right) + \pi \approx 1.9656$ or $112.6°$

$\mathbf{u} = \|\mathbf{u}\|(\cos\theta_1\,\mathbf{i} + \sin\theta_1\,\mathbf{j})$

$\mathbf{v} = \|\mathbf{v}\|(\cos\theta_2\,\mathbf{i} + \sin\theta_2\,\mathbf{j})$

Vertical components: $\|\mathbf{u}\|\sin\theta_1 + \|\mathbf{v}\|\sin\theta_2 = 5000$

Horizontal components: $\|\mathbf{u}\|\cos\theta_1 + \|\mathbf{v}\|\cos\theta_2 = 0$

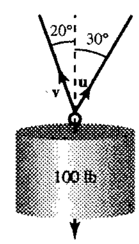
Solving this system, you obtain

$\|\mathbf{u}\| \approx 2169.4$ and $\|\mathbf{v}\| \approx 3611.2$.

91. Horizontal component $= \|\mathbf{v}\|\cos\theta = 1200\cos 6° \approx 1193.43$ ft/sec

Vertical component $= \|\mathbf{v}\|\sin\theta = 1200\sin 6° \approx 125.43$ ft/sec

92. To lift the weight vertically, the sum of the vertical components of **u** and **v** must be 100 and the sum of the horizontal components must be 0.



$\mathbf{u} = \|\mathbf{u}\|(\cos 60°\mathbf{i} + \sin 60°\mathbf{j})$

$\mathbf{v} = \|\mathbf{v}\|(\cos 110°\mathbf{i} + \sin 110°\mathbf{j})$

Thus, $\|\mathbf{u}\|\sin 60° + \|\mathbf{v}\|\sin 110° = 100$, or

$$\|\mathbf{u}\|\left(\frac{\sqrt{3}}{2}\right) + \|\mathbf{v}\|\sin 110° = 100.$$

And $\|\mathbf{u}\|\cos 60° + \|\mathbf{v}\|\cos 110° = 0$ or

$$\|\mathbf{u}\|\left(\frac{1}{2}\right) + \|\mathbf{v}\|\cos 110° = 0.$$

Multiplying the last equation by $(\sqrt{3})$ and adding to the first equation gives

$$\|\mathbf{u}\|(\sin 110° - \sqrt{3}\cos 110°) = 100 \implies \|\mathbf{v}\| \approx 65.27 \text{ lb}.$$

Then, $\|\mathbf{u}\|\left(\frac{1}{2}\right) + 65.27\cos 110° = 0$ gives

$\|\mathbf{u}\| \approx 44.65$ lb.

(a) The tension in each rope: $\|\mathbf{u}\| = 44.65$ lb, $\|\mathbf{v}\| = 65.27$ lb

(b) Vertical components: $\|\mathbf{u}\|\sin 60° \approx 38.67$ lb

$\|\mathbf{v}\|\sin 110° \approx 61.33$ lb

93. $\mathbf{u} = 900[\cos 148°\,\mathbf{i} + \sin 148°\,\mathbf{j}]$

$\mathbf{v} = 100[\cos 45°\,\mathbf{i} + \sin 45°\,\mathbf{j}]$

$\mathbf{u} + \mathbf{v} = [900\cos 148° + 100\cos 45°]\mathbf{i} + [900\sin 148° + 100\sin 45°]\mathbf{j}$

$\approx -692.53\,\mathbf{i} + 547.64\,\mathbf{j}$

$\theta \approx \arctan\left(\frac{547.64}{-692.53}\right) \approx -38.34°$; 38.34° North of West

$\|\mathbf{u} + \mathbf{v}\| \approx \sqrt{(-692.53)^2 + (547.64)^2} \approx 882.9$ km/hr

94. $\mathbf{u} = 400\mathbf{i}$(plane)

$$\mathbf{v} = 50(\cos 135°\mathbf{i} + \sin 135°\mathbf{j}) = -25\sqrt{2}\mathbf{i} + 25\sqrt{2}\mathbf{j} \text{ (wind)}$$

$$\mathbf{u} + \mathbf{v} = \left(400 - 25\sqrt{2}\right)\mathbf{i} + 25\sqrt{2}\mathbf{j} \approx 364.64\mathbf{i} + 35.36\mathbf{j}$$

$$\tan\theta = \frac{35.36}{364.64} \implies \theta \approx 5.54°$$

Direction North of East: $\approx$ N 84.46° E

Speed: $\approx$ 336.35 mph

95. True

96. True

97. True

98. False

$a = b = 0$

99. False

$\|a\mathbf{i} + b\mathbf{j}\| = \sqrt{2}\,|a|$

100. True

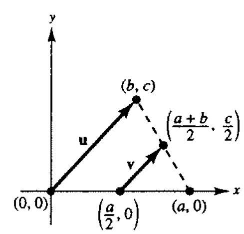
101. $\|\mathbf{u}\| = \sqrt{\cos^2\theta + \sin^2\theta} = 1,$

$\|\mathbf{v}\| = \sqrt{\sin^2\theta + \cos^2\theta} = 1$

102. Let the triangle have vertices at $(0, 0)$, $(a, 0)$, and (b, c). Let $\mathbf{u}$ be the vector joining $(0, 0)$ and (b, c), as indicated in the figure. Then $\mathbf{v}$, the vector joining the midpoints, is

$$\mathbf{v} = \left(\frac{a+b}{2} - \frac{a}{2}\right)\mathbf{i} + \frac{c}{2}\mathbf{j}$$

$$= \frac{b}{2}\mathbf{i} + \frac{c}{2}\mathbf{j} = \frac{1}{2}(b\mathbf{i} + c\mathbf{j}) = \frac{1}{2}\mathbf{u}.$$

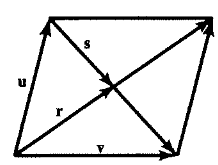


103. Let $\mathbf{u}$ and $\mathbf{v}$ be the vectors that determine the parallelogram, as indicated in the figure. The two diagonals are $\mathbf{u} + \mathbf{v}$ and $\mathbf{v} - \mathbf{u}$. Therefore, $\mathbf{r} = x(\mathbf{u} + \mathbf{v})$, $\mathbf{s} = y(\mathbf{v} - \mathbf{u})$. But,

$$\mathbf{u} = \mathbf{r} - \mathbf{s}$$

$$= x(\mathbf{u} + \mathbf{v}) - y(\mathbf{v} - \mathbf{u}) = (x + y)\mathbf{u} + (x - y)\mathbf{v}.$$

Therefore, $x + y = 1$ and $x - y = 0$. Solving we have $x = y = \frac{1}{2}$.



104. $\mathbf{w} = \|\mathbf{u}\|\mathbf{v} + \|\mathbf{v}\|\mathbf{u}$

$$= \|\mathbf{u}\|\left[\|\mathbf{v}\|\cos\theta_\mathbf{v}\mathbf{i} + \|\mathbf{v}\|\sin\theta_\mathbf{v}\mathbf{j}\right] + \|\mathbf{v}\|\left[\|\mathbf{u}\|\cos\theta_\mathbf{u}\mathbf{i} + \|\mathbf{u}\|\sin\theta_\mathbf{u}\mathbf{j}\right] = \|\mathbf{u}\|\,\|\mathbf{v}\|\left[(\cos\theta_\mathbf{u} + \cos\theta_\mathbf{v})\mathbf{i} + (\sin\theta_\mathbf{u} + \sin\theta_\mathbf{v})\mathbf{j}\right]$$

$$= 2\|\mathbf{u}\|\,\|\mathbf{v}\|\left[\cos\left(\frac{\theta_\mathbf{u} + \theta_\mathbf{v}}{2}\right)\cos\left(\frac{\theta_\mathbf{u} - \theta_\mathbf{v}}{2}\right)\mathbf{i} + \sin\left(\frac{\theta_\mathbf{u} + \theta_\mathbf{v}}{2}\right)\cos\left(\frac{\theta_\mathbf{u} - \theta_\mathbf{v}}{2}\right)\mathbf{j}\right]$$

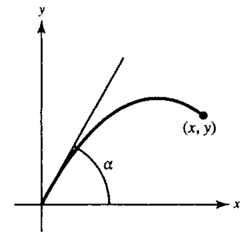
$$\tan\theta_\mathbf{w} = \frac{\sin\left(\frac{\theta_\mathbf{u} + \theta_\mathbf{v}}{2}\right)\cos\left(\frac{\theta_\mathbf{u} - \theta_\mathbf{v}}{2}\right)}{\cos\left(\frac{\theta_\mathbf{u} + \theta_\mathbf{v}}{2}\right)\cos\left(\frac{\theta_\mathbf{u} - \theta_\mathbf{v}}{2}\right)} = \tan\left(\frac{\theta_\mathbf{u} + \theta_\mathbf{v}}{2}\right)$$

Thus, $\theta_\mathbf{w} = (\theta_\mathbf{u} + \theta_\mathbf{v})/2$ and $\mathbf{w}$ bisects the angle between $\mathbf{u}$ and $\mathbf{v}$.

105. The set is a circle of radius 5, centered at the origin.

$$\|\mathbf{u}\| = \|\langle x, y\rangle\| = \sqrt{x^2 + y^2} = 5 \implies x^2 + y^2 = 25$$

106. Let $x = v_0 t \cos \alpha$ and $y = v_0 t \sin \alpha - \frac{1}{2}gt^2$.



$$t = \frac{x}{v_0 \cos \alpha} \Rightarrow y = v_0 \sin \alpha\left(\frac{x}{v_0 \cos \alpha}\right) - \frac{1}{2}g\left(\frac{x}{v_0 \cos \alpha}\right)^2$$

$$= x \tan \alpha - \frac{g}{2v_0^2}x^2 \sec^2\alpha$$

$$= x \tan \alpha - \frac{gx^2}{2v_0^2}(1 + \tan^2\alpha)$$

$$= \frac{v_0^2}{2g} - \frac{gx^2}{2v_0^2} - \frac{gx^2}{2v_0^2}\tan^2\alpha + x \tan \alpha - \frac{v_0^2}{2g}$$

$$= \frac{v_0^2}{2g} - \frac{gx^2}{2v_0^2} - \frac{gx^2}{2v_0^2}\left[\tan^2\alpha - 2 \tan \alpha\left(\frac{v_0^2}{gx}\right) + \frac{v_0^4}{g^2x^2}\right]$$

$$= \frac{v_0^2}{2g} - \frac{gx^2}{2v_0^2} - \frac{gx^2}{2v_0^2}\left(\tan \alpha - \frac{v_0^2}{gx}\right)^2$$

If $y \le \frac{v_0^2}{2g} - \frac{gx^2}{2v_0^2}$, then α can be chosen to hit the point (x, y). To hit $(0, y)$: Let $\alpha = 90°$. Then

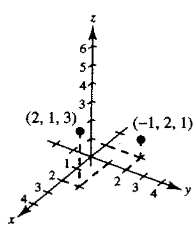
$$y = v_0 t - \frac{1}{2}gt^2 = \frac{v_0^2}{2g} - \frac{v_0^2}{2g}\left(\frac{g}{v_0}t - 1\right)^2, \text{ and you need } y \le \frac{v_0^2}{2g}.$$

The set H is given by $0 \le x, 0 < y$ and $y \le \frac{v_0^2}{2g} - \frac{gx^2}{2v_0^2}$

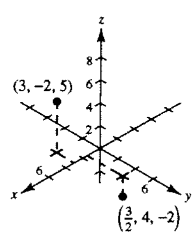
Note: The parabola $y = \frac{v_0^2}{2g} - \frac{gx^2}{2v_0^2}$ is called the "parabola of safety".

Section 11.2 Space Coordinates and Vectors in Space

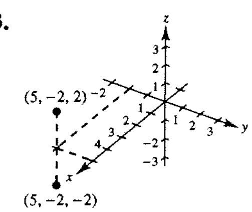
1.



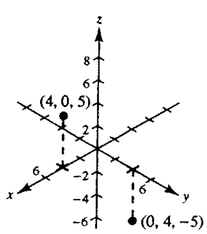
2.



3.



4.



5. $A(2, 3, 4)$

$B(-1, -2, 2)$

6. $A(2, -3, -1)$

$B(-3, 1, 4)$

7. $x = -3, y = 4, z = 5$: $(-3, 4, 5)$

8. $x = 7, y = -2, z = -1$: $(7, -2, -1)$

9. $y = z = 0, x = 10$: $(10, 0, 0)$

10. $x = 0, y = 3, z = 2$: $(0, 3, 2)$

11. The z-coordinate is 0.

12. The x-coordinate is 0.

13. The point is 6 units above the xy-plane.

14. The point is 2 units in front of the xz-plane.

15. The point is on the plane parallel to the yz-plane that passes through $x = 4$.

16. The point is on the plane $z = -3$.

17. The point is to the left of the xz-plane.

18. The point is behind the yz-plane.

19. The point is on or between the planes $y = 3$ and $y = -3$.

20. The point is in front of the plane $x = 4$.

21. The point (x, y, z) is 3 units below the xy-plane, and below either quadrant I or III.

22. The point (x, y, z) is 4 units above the xy-plane, and above either quadrant II or IV.

23. The point could be above the xy-plane and thus above quadrants II or IV, or below the xy-plane, and thus below quadrants I or III.

24. The point could be above the xy-plane, and thus above quadrants I or III, or below the xy-plane, and thus below quadrants II or IV.

25. $d = \sqrt{(5-0)^2 + (2-0)^2 + (6-0)^2}$

$= \sqrt{25 + 4 + 36} = \sqrt{65}$

26. $d = \sqrt{(2-(-2))^2 + (-5-3)^2 + (-2-2)^2}$

$= \sqrt{16 + 64 + 16} = \sqrt{96} = 4\sqrt{6}$

27. $d = \sqrt{(6-1)^2 + (-2-(-2))^2 + (-2-4)^2}$

$= \sqrt{25 + 0 + 36} = \sqrt{61}$

28. $d = \sqrt{(4-2)^2 + (-5-2)^2 + (6-3)^2}$

$= \sqrt{4 + 49 + 9} = \sqrt{62}$

29. $A(0, 0, 0), B(2, 2, 1), C(2, -4, 4)$

$|AB| = \sqrt{4 + 4 + 1} = 3$

$|AC| = \sqrt{4 + 16 + 16} = 6$

$|BC| = \sqrt{0 + 36 + 9} = 3\sqrt{5}$

$|BC|^2 = |AB|^2 + |AC|^2$

Right triangle

30. $A(5, 3, 4), B(7, 1, 3), C(3, 5, 3)$

$|AB| = \sqrt{4 + 4 + 1} = 3$

$|AC| = \sqrt{4 + 4 + 1} = 3$

$|BC| = \sqrt{16 + 16 + 0} = 4\sqrt{2}$

Since $|AB| = |AC|$, the triangle is isosceles.

31. $A(1, -3, -2), B(5, -1, 2), C(-1, 1, 2)$

$|AB| = \sqrt{16 + 4 + 16} = 6$

$|AC| = \sqrt{4 + 16 + 16} = 6$

$|BC| = \sqrt{36 + 4 + 0} = 2\sqrt{10}$

Since $|AB| = |AC|$, the triangle is isosceles.

32. $A(5, 0, 0), B(0, 2, 0), C(0, 0, -3)$

$|AB| = \sqrt{25 + 4 + 0} = \sqrt{29}$

$|AC| = \sqrt{25 + 0 + 9} = \sqrt{34}$

$|BC| = \sqrt{0 + 4 + 9} = \sqrt{13}$

Neither

33. The z-coordinate is changed by 5 units:

$(0, 0, 5), (2, 2, 6), (2, -4, 9)$

34. The y-coordinate is changed by 3 units:

$(5, 6, 4), (7, 4, 3), (3, 8, 3)$

35. $\left(\frac{5 + (-2)}{2}, \frac{-9 + 3}{2}, \frac{7 + 3}{2}\right) = \left(\frac{3}{2}, -3, 5\right)$

36. $\left(\frac{4 + 8}{2}, \frac{0 + 8}{2}, \frac{-6 + 20}{2}\right) = (6, 4, 7)$

37. Center: $(0, 2, 5)$

Radius: 2

$(x - 0)^2 + (y - 2)^2 + (z - 5)^2 = 4$

$x^2 + y^2 + z^2 - 4y - 10z + 25 = 0$

38. Center: $(4, -1, 1)$

Radius: 5

$(x - 4)^2 + (y + 1)^2 + (z - 1)^2 = 25$

$x^2 + y^2 + z^2 - 8x + 2y - 2z - 7 = 0$

39. Center: $\dfrac{(2, 0, 0) + (0, 6, 0)}{2} = (1, 3, 0)$

Radius: $\sqrt{10}$

$(x - 1)^2 + (y - 3)^2 + (z - 0)^2 = 10$

$x^2 + y^2 + z^2 - 2x - 6y = 0$

40. Center: $(-3, 2, 4)$

$r = 3$

(tangent to yz-plane)

$(x + 3)^2 + (y - 2)^2 + (z - 4)^2 = 9$

41.

$$x^2 + y^2 + z^2 - 2x + 6y + 8z + 1 = 0$$
$$(x^2 - 2x + 1) + (y^2 + 6y + 9) + (z^2 + 8z + 16) = -1 + 1 + 9 + 16$$
$$(x - 1)^2 + (y + 3)^2 + (z + 4)^2 = 25$$

Center: $(1, -3, -4)$

Radius: 5

42.

$$x^2 + y^2 + z^2 + 9x - 2y + 10z + 19 = 0$$
$$\left(x^2 + 9x + \frac{81}{4}\right) + (y^2 - 2y + 1) + (z^2 + 10z + 25) = -19 + \frac{81}{4} + 1 + 25$$
$$\left(x + \frac{9}{2}\right)^2 + (y - 1)^2 + (z + 5)^2 = \frac{109}{4}$$

Center: $\left(-\frac{9}{2}, 1, -5\right)$

Radius: $\dfrac{\sqrt{109}}{2}$

43.

$$9x^2 + 9y^2 + 9z^2 - 6x + 18y + 1 = 0$$
$$x^2 + y^2 + z^2 - \frac{2}{3}x + 2y + \frac{1}{9} = 0$$
$$\left(x^2 - \frac{2}{3}x + \frac{1}{9}\right) + (y^2 + 2y + 1) + z^2 = -\frac{1}{9} + \frac{1}{9} + 1$$
$$\left(x - \frac{1}{3}\right)^2 + (y + 1)^2 + (z - 0)^2 = 1$$

Center: $\left(\frac{1}{3}, -1, 0\right)$

Radius: 1

44.

$$4x^2 + 4y^2 + 4z^2 - 4x - 32y + 8z + 33 = 0$$
$$x^2 + y^2 + z^2 - x - 8y + 2z + \frac{33}{4} = 0$$
$$\left(x^2 - x + \frac{1}{4}\right) + (y^2 - 8y + 16) + (z^2 + 2z + 1) = -\frac{33}{4} + \frac{1}{4} + 16 + 1$$
$$\left(x - \frac{1}{2}\right)^2 + (y - 4)^2 + (z + 1)^2 = 9$$

Center: $\left(\frac{1}{2}, 4, -1\right)$

Radius: 3

45. $x^2 + y^2 + z^2 \le 36$

Solid ball of radius 6 centered at origin.

46. $x^2 + y^2 + z^2 > 4$

Set of all points in space outside the ball of radius 2 centered at the origin.

47.

$$x^2 + y^2 + z^2 < 4x - 6y + 8z - 13$$
$$(x^2 - 4x + 4) + (y^2 + 6y + 9) + (z^2 - 8z + 16) < 4 + 9 + 16 - 13$$
$$(x - 2)^2 + (y + 3)^2 + (z - 4)^2 < 16$$

Interior of sphere of radius 4 centered at $(2, -3, 4)$.

48.

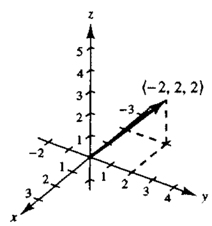
$$x^2 + y^2 + z^2 > -4x + 6y - 8z - 13$$
$$(x^2 + 4x + 4) + (y^2 - 6y + 9) + (z^2 + 8z + 16) > -13 + 4 + 9 + 16$$
$$(x + 2)^2 + (y - 3)^2 + (z + 4)^2 > 16$$

Set of all points in space outside the ball of radius 4 centered at $(-2, 3, -4)$.

49. (a) $\mathbf{v} = (2 - 4)\mathbf{i} + (4 - 2)\mathbf{j} + (3 - 1)\mathbf{k}$

$= -2\mathbf{i} + 2\mathbf{j} + 2\mathbf{k} = \langle -2, 2, 2\rangle$

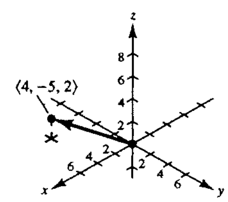
(b)



50. (a) $\mathbf{v} = (4 - 0)\mathbf{i} + (0 - 5)\mathbf{j} + (3 - 1)\mathbf{k}$

$= 4\mathbf{i} - 5\mathbf{j} + 2\mathbf{k} = \langle 4, -5, 2\rangle$

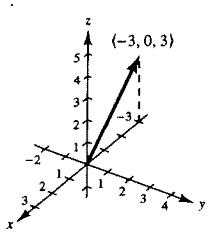
(b)



51. (a) $\mathbf{v} = (0 - 3)\mathbf{i} + (3 - 3)\mathbf{j} + (3 - 0)\mathbf{k}$

$= -3\mathbf{i} + 3\mathbf{k} = \langle -3, 0, 3\rangle$

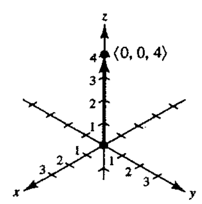
(b)



52. (a) $\mathbf{v} = (2 - 2)\mathbf{i} + (3 - 3)\mathbf{j} + (4 - 0)\mathbf{k}$

$= 4\mathbf{k} = \langle 0, 0, 4\rangle$

(b)



53. $\langle 4 - 3, 1 - 2, 6 - 0\rangle = \langle 1, -1, 6\rangle$

$\|\langle 1, -1, 6\rangle\| = \sqrt{1 + 1 + 36} = \sqrt{38}$

Unit vector: $\dfrac{\langle 1, -1, 6\rangle}{\sqrt{38}} = \left\langle \dfrac{1}{\sqrt{38}}, \dfrac{-1}{\sqrt{38}}, \dfrac{6}{\sqrt{38}}\right\rangle$

54. $\langle -1 - 4, 7 - (-5), -3 - 2\rangle = \langle -5, 12, -5\rangle$

$\|\langle -5, 12, -5\rangle\| = \sqrt{25 + 144 + 25} = \sqrt{194}$

Unit vector: $\dfrac{\langle -5, 12, -5\rangle}{\sqrt{194}} = \left\langle \dfrac{-5}{\sqrt{194}}, \dfrac{12}{\sqrt{194}}, \dfrac{-5}{\sqrt{194}}\right\rangle$

55. $\langle -5 - (-4), 3 - 3, 0 - 1\rangle = \langle -1, 0, -1\rangle$

$\|\langle -1, 0, -1\rangle\| = \sqrt{1 + 1} = \sqrt{2}$

Unit vector: $\left\langle \dfrac{-1}{\sqrt{2}}, 0, \dfrac{-1}{\sqrt{2}}\right\rangle$

56. $\langle 2 - 1, 4 - (-2), -2 - 4\rangle = \langle 1, 6, -6\rangle$

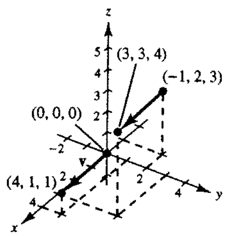
$\|\langle 1, 6, -6\rangle\| = \sqrt{1 + 36 + 36} = \sqrt{73}$

Unit vector: $\left\langle \dfrac{1}{\sqrt{73}}, \dfrac{6}{\sqrt{73}}, \dfrac{-6}{\sqrt{73}}\right\rangle$

57. (b) $\mathbf{v} = (3+1)\mathbf{i} + (3-2)\mathbf{j} + (4-3)\mathbf{k}$

$= 4\mathbf{i} + \mathbf{j} + \mathbf{k} = \langle 4, 1, 1\rangle$

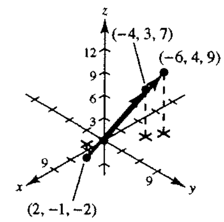
(a) and (c).



58. (b) $\mathbf{v} = (-4-2)\mathbf{i} + (3+1)\mathbf{j} + (7+2)\mathbf{k}$

$= -6\mathbf{i} + 4\mathbf{j} + 9\mathbf{k} = \langle -6, 4, 9\rangle$

(a) and (c).



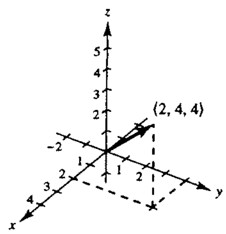
59. $(q_1, q_2, q_3) - (0, 6, 2) = (3, -5, 6)$

$Q = (3, 1, 8)$

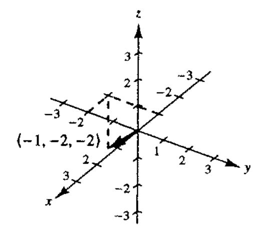
60. $(q_1, q_2, q_3) - \left(0, 2, \frac{5}{2}\right) = \left(1, -\frac{2}{3}, \frac{1}{2}\right)$

$Q = \left(1, -\frac{4}{3}, 3\right)$

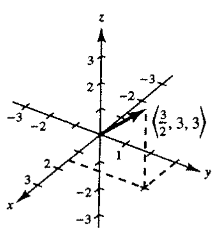
61. (a) $2\mathbf{v} = \langle 2, 4, 4\rangle$



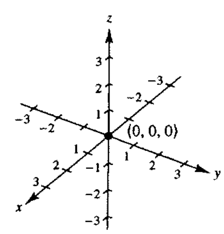
(b) $-\mathbf{v} = \langle -1, -2, -2\rangle$



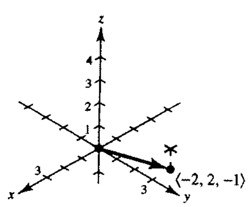
(c) $\frac{3}{2}\mathbf{v} = \left\langle \frac{3}{2}, 3, 3\right\rangle$



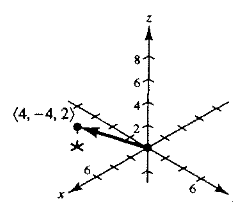
(d) $0\mathbf{v} = \langle 0, 0, 0\rangle$



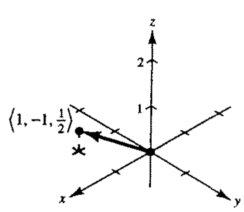
62. (a) $-\mathbf{v} = \langle -2, 2, -1\rangle$



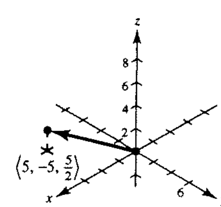
(b) $2\mathbf{v} = \langle 4, -4, 2\rangle$



(c) $\frac{1}{2}\mathbf{v} = \left\langle 1, -1, \frac{1}{2}\right\rangle$



(d) $\frac{5}{2}\mathbf{v} = \left\langle 5, -5, \frac{5}{2}\right\rangle$



63. $\mathbf{z} = \mathbf{u} - \mathbf{v} = \langle 1, 2, 3\rangle - \langle 2, 2, -1\rangle = \langle -1, 0, 4\rangle$

64. $\mathbf{z} = \mathbf{u} - \mathbf{v} + 2\mathbf{w} = \langle 1, 2, 3\rangle - \langle 2, 2, -1\rangle + \langle 8, 0, -8\rangle = \langle 7, 0, -4\rangle$

65. $\mathbf{z} = 2\mathbf{u} + 4\mathbf{v} - \mathbf{w} = \langle 2, 4, 6\rangle + \langle 8, 8, -4\rangle - \langle 4, 0, -4\rangle = \langle 6, 12, 6\rangle$

66. $\mathbf{z} = 5\mathbf{u} - 3\mathbf{v} - \frac{1}{2}\mathbf{w} = \langle 5, 10, 15\rangle - \langle 6, 6, -3\rangle - \langle 2, 0, -2\rangle = \langle -3, 4, 20\rangle$

67. $2\mathbf{z} - 3\mathbf{u} = 2\langle z_1, z_2, z_3\rangle - 3\langle 1, 2, 3\rangle = \langle 4, 0, -4\rangle$

$2z_1 - 3 = 4 \implies z_1 = \frac{7}{2}$

$2z_2 - 6 = 0 \implies z_2 = 3$

$2z_3 - 9 = -4 \implies z_3 = \frac{5}{2}$

$\mathbf{z} = \left\langle \frac{7}{2}, 3, \frac{5}{2}\right\rangle$

68. $2\mathbf{u} + \mathbf{v} - \mathbf{w} + 3\mathbf{z} = 2\langle 1, 2, 3\rangle + \langle 2, 2, -1\rangle - \langle 4, 0, -4\rangle + 3\langle z_1, z_2, z_3\rangle = \langle 0, 0, 0\rangle$

$\langle 0, 6, 9\rangle + \langle 3z_1, 3z_2, 3z_3\rangle = \langle 0, 0, 0\rangle$

$0 + 3z_1 = 0 \implies z_1 = 0$

$6 + 3z_2 = 0 \implies z_2 = -2$

$9 + 3z_3 = 0 \implies z_3 = -3$

$\mathbf{z} = \langle 0, -2, -3\rangle$

69. (a) and (b) are parallel since $\langle -6, -4, 10\rangle = -2\langle 3, 2, -5\rangle$ and $\left\langle 2, \frac{4}{3}, -\frac{10}{3}\right\rangle = \frac{2}{3}\langle 3, 2, -5\rangle$.

70. (b) and (d) are parallel since $-\mathbf{i} + \frac{4}{3}\mathbf{j} - \frac{3}{2}\mathbf{k} = -2\left(\frac{1}{2}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{3}{4}\mathbf{k}\right)$ and $\frac{3}{4}\mathbf{i} - \mathbf{j} + \frac{9}{8}\mathbf{k} = \frac{3}{2}\left(\frac{1}{2}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{3}{4}\mathbf{k}\right)$.

71. $\mathbf{z} = -3\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$

(a) is parallel since $-6\mathbf{i} + 8\mathbf{j} + 4\mathbf{k} = 2\mathbf{z}$.

72. $\mathbf{z} = \langle -7, -8, 3\rangle$

(b) is parallel since $(-z)\mathbf{z} = \langle 14, 16, -6\rangle$.

73. $P(0, -2, -5), Q(3, 4, 4), R(2, 2, 1)$

$\overrightarrow{PQ} = \langle 3, 6, 9\rangle$

$\overrightarrow{PR} = \langle 2, 4, 6\rangle$

$\langle 3, 6, 9\rangle = \frac{3}{2}\langle 2, 4, 6\rangle$

Therefore, $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are parallel. The points are collinear.

74. $P(4, -2, 7), Q(-2, 0, 3), R(7, -3, 9)$

$\overrightarrow{PQ} = \langle -6, 2, -4\rangle$

$\overrightarrow{PR} = \langle 3, -1, 2\rangle$

$\langle 3, -1, 2\rangle = -\frac{1}{2}\langle -6, 2, -4\rangle$

Therefore, $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are parallel. The points are collinear.

75. $P(1, 2, 4), Q(2, 5, 0), R(0, 1, 5)$

$\overrightarrow{PQ} = \langle 1, 3, -4\rangle$

$\overrightarrow{PR} = \langle -1, -1, 1\rangle$

Since $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are not parallel, the points are not collinear.

76. $P(0, 0, 0), Q(1, 3, -2), R(2, -6, 4)$

$\overrightarrow{PQ} = \langle 1, 3, -2\rangle$

$\overrightarrow{PR} = \langle 2, -6, 4\rangle$

Since $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are not parallel, the points are not collinear.

77. $A(2, 9, 1), B(3, 11, 4), C(0, 10, 2), D(1, 12, 5)$

$\overrightarrow{AB} = \langle 1, 2, 3\rangle$

$\overrightarrow{CD} = \langle 1, 2, 3\rangle$

$\overrightarrow{AC} = \langle -2, 1, 1\rangle$

$\overrightarrow{BD} = \langle -2, 1, 1\rangle$

Since $\overrightarrow{AB} = \overrightarrow{CD}$ and $\overrightarrow{AC} = \overrightarrow{BD}$, the given points form the vertices of a parallelogram.

78. $A(1, 1, 3), B(9, -1, -2), C(11, 2, -9), D(3, 4, -4)$

$\overrightarrow{AB} = \langle 8, -2, -5\rangle$

$\overrightarrow{DC} = \langle 8, -2, -5\rangle$

$\overrightarrow{AD} = \langle 2, 3, -7\rangle$

$\overrightarrow{BC} = \langle 2, 3, -7\rangle$

Since $\overrightarrow{AB} = \overrightarrow{DC}$ and $\overrightarrow{AD} = \overrightarrow{BC}$, the given points form the vertices of a parallelogram.

79. $\|\mathbf{v}\| = 0$

80. $\|\mathbf{v}\| = \sqrt{1 + 0 + 9} = \sqrt{10}$

81. $\mathbf{v} = \langle 1, -2, -3\rangle$

$\|\mathbf{v}\| = \sqrt{1 + 4 + 9} = \sqrt{14}$

82. $\mathbf{v} = \langle -4, 3, 7\rangle$

$\|\mathbf{v}\| = \sqrt{16 + 9 + 49} = \sqrt{74}$

83. $\mathbf{v} = \langle 0, 3, -5\rangle$

$\|\mathbf{v}\| = \sqrt{0 + 9 + 25} = \sqrt{34}$

84. $\mathbf{v} = \langle 1, 3, -2\rangle$

$\|\mathbf{v}\| = \sqrt{1 + 9 + 4} = \sqrt{14}$

85. $\mathbf{u} = \langle 2, -1, 2\rangle$

$\|\mathbf{u}\| = \sqrt{4 + 1 + 4} = 3$

(a) $\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = \dfrac{1}{3}\langle 2, -1, 2\rangle$

(b) $-\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = -\dfrac{1}{3}\langle 2, -1, 2\rangle$

86. $\mathbf{u} = \langle 6, 0, 8\rangle$

$\|\mathbf{u}\| = \sqrt{36 + 0 + 64} = 10$

(a) $\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = \dfrac{1}{10}\langle 6, 0, 8\rangle$

(b) $-\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = -\dfrac{1}{10}\langle 6, 0, 8\rangle$

87. $\mathbf{u} = \langle 3, 2, -5\rangle$

$\|\mathbf{u}\| = \sqrt{9 + 4 + 25} = \sqrt{38}$

(a) $\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = \dfrac{1}{\sqrt{38}}\langle 3, 2, -5\rangle$

(b) $-\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = -\dfrac{1}{\sqrt{38}}\langle 3, 2, -5\rangle$

88. $\mathbf{u} = \langle 8, 0, 0\rangle$

$\|\mathbf{u}\| = 8$

(a) $\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = \langle 1, 0, 0\rangle$

(b) $-\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = \langle -1, 0, 0\rangle$

89. Programs will vary.

90. (a) $\mathbf{u} + \mathbf{v} = \langle 4, 7.5, -2\rangle$

(b) $\|\mathbf{u} + \mathbf{v}\| \approx 8.732$

(c) $\|\mathbf{u}\| \approx 5.099$

(d) $\|\mathbf{v}\| \approx 9.014$

91. $c\mathbf{v} = \langle 2c, 2c, -c\rangle$

$\|c\mathbf{v}\| = \sqrt{4c^2 + 4c^2 + c^2} = 5$

$9c^2 = 25$

$c = \pm\dfrac{5}{3}$

92. $c\mathbf{u} = \langle c, 2c, 3c\rangle$

$\|c\mathbf{u}\| = \sqrt{c^2 + 4c^2 + 9c^2} = 3$

$14c^2 = 9$

$c = \pm\dfrac{3\sqrt{14}}{14}$

93. $\mathbf{v} = 10\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = 10\left\langle 0, \dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right\rangle$

$= \left\langle 0, \dfrac{10}{\sqrt{2}}, \dfrac{10}{\sqrt{2}}\right\rangle$

94. $\mathbf{v} = 3\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = 3\left\langle \dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}\right\rangle = \left\langle \dfrac{3}{\sqrt{3}}, \dfrac{3}{\sqrt{3}}, \dfrac{3}{\sqrt{3}}\right\rangle$

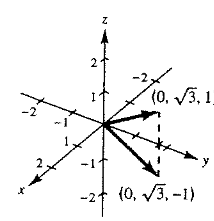
95. $\mathbf{v} = \dfrac{3}{2}\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = \dfrac{3}{2}\left\langle \dfrac{2}{3}, \dfrac{-2}{3}, \dfrac{1}{3}\right\rangle = \left\langle 1, -1, \dfrac{1}{2}\right\rangle$

96. $\mathbf{v} = \sqrt{5}\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = \sqrt{5}\left\langle \dfrac{-2}{\sqrt{14}}, \dfrac{3}{\sqrt{14}}, \dfrac{1}{\sqrt{14}}\right\rangle$

$= \left\langle \dfrac{-\sqrt{70}}{7}, \dfrac{3\sqrt{70}}{14}, \dfrac{\sqrt{70}}{14}\right\rangle$

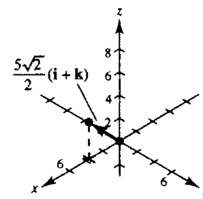
97. $\mathbf{v} = 2[\cos(\pm 30°)\mathbf{j} + \sin(\pm 30°)\mathbf{k}]$

$= \sqrt{3}\mathbf{j} \pm \mathbf{k} = \left\langle 0, \sqrt{3}, \pm 1\right\rangle$



98. $\mathbf{v} = 5(\cos 45°\mathbf{i} + \sin 45°\mathbf{k}) = \dfrac{5\sqrt{2}}{2}(\mathbf{i} + \mathbf{k})$ or

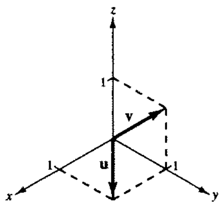
$\mathbf{v} = 5(\cos 135°\mathbf{i} + \sin 135°\mathbf{k}) = \dfrac{5\sqrt{2}}{2}(-\mathbf{i} + \mathbf{k})$



99.
$$\mathbf{v} = \langle -3, -6, 3\rangle$$
$$\tfrac{2}{3}\mathbf{v} = \langle -2, -4, 2\rangle$$
$$(4, 3, 0) + (-2, -4, 2) = (2, -1, 2)$$

100.
$$\mathbf{v} = \langle 5, 6, -3\rangle$$
$$\tfrac{2}{3}\mathbf{v} = \left\langle \tfrac{10}{3}, 4, -2\right\rangle$$
$$(1, 2, 5) + \left(\tfrac{10}{3}, 4, -2\right) = \left(\tfrac{13}{3}, 6, 3\right)$$

101. (a)



(b) $\mathbf{w} = a\mathbf{u} + b\mathbf{v} = a\mathbf{i} + (a + b)\mathbf{j} + b\mathbf{k} = \mathbf{0}$

$a = 0, a + b = 0, b = 0$

Thus, a and b are both zero.

(c) $a\mathbf{i} + (a + b)\mathbf{j} + b\mathbf{k} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$

$a = 1, b = 1$

$\mathbf{w} = \mathbf{u} + \mathbf{v}$

(d) $a\mathbf{i} + (a + b)\mathbf{j} + b\mathbf{k} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$

$a = 1, a + b = 2, b = 3$

Not possible

102. A sphere of radius 4 centered at (x_1, y_1, z_1).

$\|\mathbf{v}\| = \|\langle x - x_2, y - y_1, z - z_1\rangle\|$

$= \sqrt{(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2} = 4$

$(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 = 16$ sphere

103. x_0 is directed distance to yz-plane.

y_0 is directed distance to xz-plane.

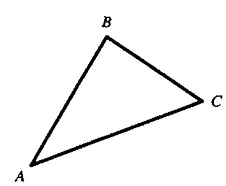
z_0 is directed distance to xy-plane.

104. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

105. $(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$

106. Two nonzero vectors $\mathbf{u}$ and $\mathbf{v}$ are parallel if $\mathbf{u} = c\mathbf{v}$ for some scalar c.

107.



$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

Hence, $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{AC} + \overrightarrow{CA} = \mathbf{0}$

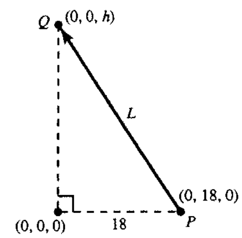
108. $\|\mathbf{r} - \mathbf{r}_0\| = \sqrt{(x - 1)^2 + (y - 1)^2 + (z - 1)^2} = 2$

$(x - 1)^2 + (y - 1)^2 + (z - 1)^2 = 4$

This is a sphere of radius 2 and center $(1, 1, 1)$.

109. (a) The height of the right triangle is $h = \sqrt{L^2 - 18^2}$.
The vector $\overrightarrow{PQ}$ is given by

$$\overrightarrow{PQ} = \langle 0, -18, h\rangle.$$



The tension vector **T** in each wire is

$$\mathbf{T} = c\langle 0, -18, h\rangle \text{ where } ch = \frac{24}{3} = 8.$$

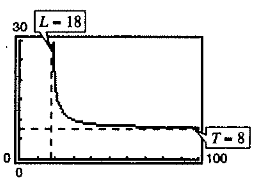
Hence, $\mathbf{T} = \dfrac{8}{h}\langle 0, -18, h\rangle$ and

$$T = \|\mathbf{T}\| = \frac{8}{h}\sqrt{18^2 + h^2} = \frac{8}{\sqrt{L^2 - 18^2}}\sqrt{18^2 + (L^2 - 18^2)} = \frac{8L}{\sqrt{L^2 - 18^2}}.$$

(b)

L	20	25	30	35	40	45	50
T	18.4	11.5	10	9.3	9.0	8.7	8.6

(c)



$x = 18$ is a vertical asymptote and $y = 8$ is a horizontal asymptote.

(d) $$\lim_{L\to 18^+} \frac{8L}{\sqrt{L^2 - 18^2}} = \infty$$

$$\lim_{L\to\infty} \frac{8L}{\sqrt{L^2 - 18^2}} = \lim_{L\to\infty} \frac{8}{\sqrt{1 - (18/L)^2}} = 8$$

(e) From the table, $T = 10$ implies $L = 30$ inches.

110. As in Exercise 109(c), $x = a$ will be a vertical asymptote. Hence, $\displaystyle\lim_{r_0\to a^-} T = \infty$.

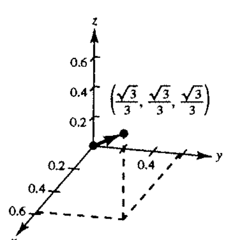
111. Let α be the angle between **v** and the coordinate axes.

$$\mathbf{v} = (\cos\alpha)\mathbf{i} + (\cos\alpha)\mathbf{j} + (\cos\alpha)\mathbf{k}$$

$$\|\mathbf{v}\| = \sqrt{3}\cos\alpha = 1$$

$$\cos\alpha = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\mathbf{v} = \frac{\sqrt{3}}{3}(\mathbf{i} + \mathbf{j} + \mathbf{k}) = \frac{\sqrt{3}}{3}\langle 1, 1, 1\rangle$$



112.

$$550 = \|c(75\mathbf{i} - 50\mathbf{j} - 100\mathbf{k})\|$$

$$302{,}500 = 18{,}125c^2$$

$$c^2 = 16.689655$$

$$c \approx 4.085$$

$$\mathbf{F} \approx 4.085(75\mathbf{i} - 50\mathbf{j} - 100\mathbf{k})$$

$$\approx 306\mathbf{i} - 204\mathbf{j} - 409\mathbf{k}$$

113. $\overrightarrow{AB} = \langle 0, 70, 115\rangle, \mathbf{F}_1 = C_1\langle 0, 70, 115\rangle$

$\overrightarrow{AC} = \langle -60, 0, 115\rangle, \mathbf{F}_2 = C_2\langle -60, 0, 115\rangle$

$\overrightarrow{AD} = \langle 45, -65, 115\rangle, \mathbf{F}_3 = C_3\langle 45, -65, 115\rangle$

$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = \langle 0, 0, 500\rangle$

Thus:

$$\begin{aligned} -60C_2 + 45C_3 &= 0 \\ 70C_1 \qquad - 65C_3 &= 0 \\ 115(C_1 + C_2 + C_3) &= 500 \end{aligned}$$

Solving this system yields $C_1 = \frac{104}{69}$, $C_2 = \frac{28}{23}$, and $C_3 = \frac{112}{69}$. Thus:

$\|\mathbf{F}_1\| \approx 202.919N$

$\|\mathbf{F}_2\| \approx 157.909N$

$\|\mathbf{F}_3\| \approx 226.521N$

114. Let A lie on the y-axis and the wall on the x-axis. Then $A = (0, 10, 0)$, $B = (8, 0, 6)$, $C = (-10, 0, 6)$ and

$\overrightarrow{AB} = \langle 8, -10, 6\rangle$, $\overrightarrow{AC} = \langle -10, -10, 6\rangle$.

$\|AB\| = 10\sqrt{2}, \|AC\| = 2\sqrt{59}$

Thus, $\mathbf{F}_1 = 420\frac{\overrightarrow{AB}}{\|\overrightarrow{AB}\|}, \mathbf{F}_2 = 650\frac{\overrightarrow{AC}}{\|\overrightarrow{AC}\|}$

$$\begin{aligned} \mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 &\approx \langle 237.6, -297.0, 178.2\rangle \\ &\quad + \langle -423.1, -423.1, 253.9\rangle \\ &\approx \langle -185.5, -720.1, 432.1\rangle \end{aligned}$$

$\|\mathbf{F}\| \approx 860.0$ lb

115. $d(AP) = 2d(BP)$

$$\begin{aligned} \sqrt{x^2 + (y+1)^2 + (z-1)^2} &= 2\sqrt{(x-1)^2 + (y-2)^2 + z^2} \\ x^2 + y^2 + z^2 + 2y - 2z + 2 &= 4(x^2 + y^2 + z^2 - 2x - 4y + 5) \\ 0 &= 3x^2 + 3y^2 + 3z^2 - 8x - 18y + 2z + 18 \\ -6 + \frac{16}{9} + 9 + \frac{1}{9} &= \left(x^2 - \frac{8}{3}x + \frac{16}{9}\right) + (y^2 - 6y + 9) + \left(z^2 + \frac{2}{3}z + \frac{1}{9}\right) \\ \frac{44}{9} &= \left(x - \frac{4}{3}\right)^2 + (y-3)^2 + \left(z + \frac{1}{3}\right)^2 \end{aligned}$$

Sphere; center: $\left(\frac{4}{3}, 3, -\frac{1}{3}\right)$, radius: $\frac{2\sqrt{11}}{3}$

Section 11.3 The Dot Product of Two Vectors

1. $\mathbf{u} = \langle 3, 4\rangle, \mathbf{v} = \langle 2, -3\rangle$

(a) $\mathbf{u} \cdot \mathbf{v} = 3(2) + 4(-3) = -6$

(b) $\mathbf{u} \cdot \mathbf{u} = 3(3) + 4(4) = 25$

(c) $\|\mathbf{u}\|^2 = 25$

(d) $(\mathbf{u} \cdot \mathbf{v})\mathbf{v} = -6\langle 2, -3\rangle = \langle -12, 18\rangle$

(e) $\mathbf{u} \cdot (2\mathbf{v}) = 2(\mathbf{u} \cdot \mathbf{v}) = 2(-6) = -12$

2. $\mathbf{u} = \langle 4, 10\rangle, \mathbf{v} = \langle -2, 3\rangle$

(a) $\mathbf{u} \cdot \mathbf{v} = 4(-2) + 10(3) = 22$

(b) $\mathbf{u} \cdot \mathbf{u} = 4(4) + 10(10) = 116$

(c) $\|\mathbf{u}\|^2 = 116$

(d) $(\mathbf{u} \cdot \mathbf{v})\mathbf{v} = 22\langle -2, 3\rangle = \langle -44, 66\rangle$

(e) $\mathbf{u} \cdot (2\mathbf{v}) = 2(\mathbf{u} \cdot \mathbf{v}) = 2(22) = 44$

3. (a) $\mathbf{u} \cdot \mathbf{v} = \langle 5, -1\rangle \cdot \langle -3, 2\rangle = 5(-3) + (-1)(2) = -17$

(b) $\mathbf{u} \cdot \mathbf{u} = \langle 5, -1\rangle \cdot \langle 5, -1\rangle = 5(5) + (-1)(-1) = 26$

(c) $\|\mathbf{u}\|^2 = \mathbf{u} \cdot \mathbf{u} = 26$

(d) $(\mathbf{u} \cdot \mathbf{v})\mathbf{v} = (-17)\langle -3, 2\rangle = \langle 51, -34\rangle$

(e) $\mathbf{u} \cdot (2\mathbf{v}) = 2(\mathbf{u} \cdot \mathbf{v}) = 2(-17) = -34$

4. (a) $\mathbf{u} \cdot \mathbf{v} = \langle -4, 8\rangle \cdot \langle 6, 3\rangle = (-4)6 + 8(3) = 0$

(b) $\mathbf{u} \cdot \mathbf{u} = \langle -4, 8\rangle \cdot \langle -4, 8\rangle = (-4)(-4) + 8(8) = 80$

(c) $\|\mathbf{u}\|^2 = \mathbf{u} \cdot \mathbf{u} = 80$

(d) $(\mathbf{u} \cdot \mathbf{v})\mathbf{v} = 0\mathbf{v} = \mathbf{0}$

(e) $\mathbf{u} \cdot (2\mathbf{v}) = 2(\mathbf{u} \cdot \mathbf{v}) = 2(0) = 0$

5. $\mathbf{u} = \langle 2, -3, 4\rangle, \mathbf{v} = \langle 0, 6, 5\rangle$

(a) $\mathbf{u} \cdot \mathbf{v} = 2(0) + (-3)(6) + (4)(5) = 2$

(b) $\mathbf{u} \cdot \mathbf{u} = 2(2) + (-3)(-3) + 4(4) = 29$

(c) $\|\mathbf{u}\|^2 = 29$

(d) $(\mathbf{u} \cdot \mathbf{v})\mathbf{v} = 2\langle 0, 6, 5\rangle = \langle 0, 12, 10\rangle$

(e) $\mathbf{u} \cdot (2\mathbf{v}) = 2(\mathbf{u} \cdot \mathbf{v}) = 2(2) = 4$

6. $\mathbf{u} = \mathbf{i}, \mathbf{v} = \mathbf{i}$

(a) $\mathbf{u} \cdot \mathbf{v} = 1$

(b) $\mathbf{u} \cdot \mathbf{u} = 1$

(c) $\|\mathbf{u}\|^2 = 1$

(d) $(\mathbf{u} \cdot \mathbf{v})\mathbf{v} = \mathbf{i}$

(e) $\mathbf{u} \cdot (2\mathbf{v}) = 2(\mathbf{u} \cdot \mathbf{v}) = 2$

7. $\mathbf{u} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}, \mathbf{v} = \mathbf{i} - \mathbf{k}$

(a) $\mathbf{u} \cdot \mathbf{v} = 2(1) + (-1)(0) + 1(-1) = 1$

(b) $\mathbf{u} \cdot \mathbf{u} = 2(2) + (-1)(-1) + (1)(1) = 6$

(c) $\|\mathbf{u}\|^2 = 6$

(d) $(\mathbf{u} \cdot \mathbf{v})\mathbf{v} = \mathbf{v} = \mathbf{i} - \mathbf{k}$

(e) $\mathbf{u} \cdot (2\mathbf{v}) = 2(\mathbf{u} \cdot \mathbf{v}) = 2$

8. $\mathbf{u} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}, \mathbf{v} = \mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$

(a) $\mathbf{u} \cdot \mathbf{v} = 2(1) + 1(-3) + (-2)(2) = -5$

(b) $\mathbf{u} \cdot \mathbf{u} = 2(2) + 1(1) + (-2)(-2) = 9$

(c) $\|\mathbf{u}\|^2 = 9$

(d) $(\mathbf{u} \cdot \mathbf{v})\mathbf{v} = -5(\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}) = -5\mathbf{i} + 15\mathbf{j} - 10\mathbf{k}$

(e) $\mathbf{u} \cdot (2\mathbf{v}) = 2(\mathbf{u} \cdot \mathbf{v}) = 2(-5) = -10$

9. $\dfrac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \cos\theta$

$$\mathbf{u} \cdot \mathbf{v} = (8)(5)\cos\frac{\pi}{3} = 20$$

10. $\dfrac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \cos\theta$

$$\mathbf{u} \cdot \mathbf{v} = (40)(25)\cos\frac{5\pi}{6} = -500\sqrt{3}$$

11. $\mathbf{u} = \langle 1, 1\rangle, \mathbf{v} = \langle 2, -2\rangle$

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{0}{\sqrt{2}\sqrt{8}} = 0$$

$$\theta = \frac{\pi}{2}$$

12. $\mathbf{u} = \langle 3, 1\rangle, \mathbf{v} = \langle 2, -1\rangle$

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{5}{\sqrt{10}\sqrt{5}} = \frac{1}{\sqrt{2}}$$

$$\theta = \frac{\pi}{4}$$

13. $\mathbf{u} = 3\mathbf{i} + \mathbf{j}, \mathbf{v} = -2\mathbf{i} + 4\mathbf{j}$

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{-2}{\sqrt{10}\sqrt{20}} = \frac{-1}{5\sqrt{2}}$$

$$\theta = \arccos\left(-\frac{1}{5\sqrt{2}}\right) \approx 98.1^\circ$$

14.

$$\mathbf{u} = \cos\left(\frac{\pi}{6}\right)\mathbf{i} + \sin\left(\frac{\pi}{6}\right)\mathbf{j} = \frac{\sqrt{3}}{2}\mathbf{i} + \frac{1}{2}\mathbf{j}$$

$$\mathbf{v} = \cos\left(\frac{3\pi}{4}\right)\mathbf{i} + \sin\left(\frac{3\pi}{4}\right)\mathbf{j} = -\frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j}$$

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|}$$

$$= \frac{\sqrt{3}}{2}\left(-\frac{\sqrt{2}}{2}\right) + \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{4}(1 - \sqrt{3})$$

$$\theta = \arccos\left[\frac{\sqrt{2}}{4}(1 - \sqrt{3})\right] = 105^\circ$$

15. $\mathbf{u} = \langle 1, 1, 1\rangle, \mathbf{v} = \langle 2, 1, -1\rangle$

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{2}{\sqrt{3}\sqrt{6}} = \frac{\sqrt{2}}{3}$$

$$\theta = \arccos\frac{\sqrt{2}}{3} \approx 61.9^\circ$$

16. $\mathbf{u} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k}, \mathbf{v} = 2\mathbf{i} - 3\mathbf{j}$

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{3(2) + 2(-3) + 0}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = 0$$

$$\theta = \frac{\pi}{2}$$

17. $\mathbf{u} = 3\mathbf{i} + 4\mathbf{j}, \mathbf{v} = -2\mathbf{j} + 3\mathbf{k}$

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{-8}{5\sqrt{13}} = \frac{-8\sqrt{13}}{65}$$

$$\theta = \arccos\left(-\frac{8\sqrt{13}}{65}\right) \approx 116.3°$$

18. $\mathbf{u} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}, \mathbf{v} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|}$$

$$= \frac{9}{\sqrt{14}\sqrt{6}} = \frac{9}{2\sqrt{21}} = \frac{3\sqrt{21}}{14}$$

$$\theta = \arccos\left(\frac{3\sqrt{21}}{14}\right) \approx 10.9°$$

19. $\mathbf{u} = \langle 4, 0\rangle, \mathbf{v} = \langle 1, 1\rangle$

$\mathbf{u} \neq c\mathbf{v} \implies$ not parallel

$\mathbf{u}\cdot\mathbf{v} = 4 \neq 0 \implies$ not orthogonal

Neither

20. $\mathbf{u} = \langle 2, 18\rangle, \mathbf{v} = \left\langle \frac{3}{2}, -\frac{1}{6}\right\rangle$

$\mathbf{u} \neq c\mathbf{v} \implies$ not parallel

$\mathbf{u}\cdot\mathbf{v} = 0 \implies$ orthogonal

21. $\mathbf{u} = \langle 4, 3\rangle, \mathbf{v} = \left\langle \frac{1}{2}, -\frac{2}{3}\right\rangle$

$\mathbf{u} \neq c\mathbf{v} \implies$ not parallel

$\mathbf{u}\cdot\mathbf{v} = 0 \implies$ orthogonal

22. $\mathbf{u} = -\frac{1}{3}(\mathbf{i} - 2\mathbf{j}), \mathbf{v} = 2\mathbf{i} - 4\mathbf{j}$

$\mathbf{u} = -\frac{1}{6}\mathbf{v} \implies$ parallel

23. $\mathbf{u} = \mathbf{j} + 6\mathbf{k}, \mathbf{v} = \mathbf{i} - 2\mathbf{j} - \mathbf{k}$

$\mathbf{u} \neq c\mathbf{v} \implies$ not parallel

$\mathbf{u}\cdot\mathbf{v} = -8 \neq 0 \implies$ not orthogonal

Neither

24. $\mathbf{u} = -2\mathbf{i} + 3\mathbf{j} - \mathbf{k}, \mathbf{v} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$

$\mathbf{u} \neq c\mathbf{v} \implies$ not parallel

$\mathbf{u}\cdot\mathbf{v} = 0 \implies$ orthogonal

25. $\mathbf{u} = \langle 2, -3, 1\rangle, \mathbf{v} = \langle -1, -1, -1\rangle$

$\mathbf{u} \neq c\mathbf{v} \implies$ not parallel

$\mathbf{u}\cdot\mathbf{v} = 0 \implies$ orthogonal

26. $\mathbf{u} = \langle \cos\theta, \sin\theta, -1\rangle$,

$\mathbf{v} = \langle \sin\theta, -\cos\theta, 0\rangle$

$\mathbf{u} \neq c\mathbf{v} \implies$ not parallel

$\mathbf{u}\cdot\mathbf{v} = 0 \implies$ orthogonal

27. The vector $\langle 1, 2, 0\rangle$ joining $(1, 2, 0)$ and $(0, 0, 0)$ is perpendicular to the vector $\langle -2, 1, 0\rangle$ joining $(-2, 1, 0)$ and $(0, 0, 0)$:

$$\langle 1, 2, 0\rangle \cdot \langle -2, 1, 0\rangle = 0$$

The triangle is a right triangle.

28. Consider the vector $\langle -3, 0, 0\rangle$ joining $(0, 0, 0)$ and $(-3, 0, 0)$, and the vector $\langle 1, 2, 3\rangle$ joining $(0, 0, 0)$ and $(1, 2, 3)$:

$$\langle -3, 0, 0\rangle \cdot \langle 1, 2, 3\rangle = -3 < 0$$

The triangle has an obtuse angle.

29. The vectors forming the sides of the triangle are:

$\mathbf{u} = \langle 0 - 2, 1 + 3, 2 - 4\rangle = \langle -2, 4, -2\rangle$

$\mathbf{v} = \langle -1 - 2, 2 + 3, 0 - 4\rangle = \langle -3, 5, -4\rangle$

$\mathbf{w} = \langle -1 - 0, 2 - 1, 0 - 2\rangle = \langle -1, 1, -2\rangle$

$\mathbf{u}\cdot\mathbf{v} = 6 + 9 + 8 = 23 > 0$

$\mathbf{u}\cdot\mathbf{w} = 2 + 4 + 4 = 10 > 0$

$\mathbf{v}\cdot\mathbf{w} = 4 + 5 + 8 = 17 > 0$

The triangle has three acute angles.

30. The vectors forming the sides of the triangle are:

$\mathbf{u} = \langle -1 - 2, 5 + 7, 8 - 3\rangle = \langle -3, 12, 5\rangle$

$\mathbf{v} = \langle 4 - 2, 6 + 7, -1 - 3\rangle = \langle 2, 13, -4\rangle$

$\mathbf{w} = \langle 4 + 1, 6 - 5, -1 - 8\rangle = \langle 5, 1, -9\rangle$

$\mathbf{u}\cdot\mathbf{w} = -15 + 12 - 45 = -48 < 0$

The triangle has an obtuse angle.

31. $\mathbf{u} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}, \|\mathbf{u}\| = 3$

$$\cos\alpha = \frac{1}{3}$$

$$\cos\beta = \frac{2}{3}$$

$$\cos\gamma = \frac{2}{3}$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{1}{9} + \frac{4}{9} + \frac{4}{9} = 1$$

32. $\mathbf{u} = \langle 5, 3, -1\rangle \quad \|\mathbf{u}\| = \sqrt{35}$

$$\cos\alpha = \frac{5}{\sqrt{35}}$$

$$\cos\beta = \frac{3}{\sqrt{35}}$$

$$\cos\gamma = \frac{-1}{\sqrt{35}}$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{25}{35} + \frac{9}{35} + \frac{1}{35} = 1$$

33. $\mathbf{u} = \langle 0, 6, -4\rangle, \|\mathbf{u}\| = \sqrt{52} = 2\sqrt{13}$

$\cos\alpha = 0$

$\cos\beta = \frac{3}{\sqrt{13}}$

$\cos\gamma = -\frac{2}{\sqrt{13}}$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 0 + \frac{9}{13} + \frac{4}{13} = 1$

34. $\mathbf{u} = \langle a, b, c\rangle, \|\mathbf{u}\| = \sqrt{a^2 + b^2 + c^2}$

$\cos\alpha = \frac{a}{\sqrt{a^2 + b^2 + c^2}}$

$\cos\beta = \frac{b}{\sqrt{a^2 + b^2 + c^2}}$

$\cos\gamma = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{a^2}{a^2 + b^2 + c^2} + \frac{b^2}{a^2 + b^2 + c^2} + \frac{c^2}{a^2 + b^2 + c^2} = 1$

35. $\mathbf{u} = \langle 3, 2, -2\rangle \quad \|\mathbf{u}\| = \sqrt{17}$

$\cos\alpha = \frac{3}{\sqrt{17}} \implies \alpha \approx 0.7560 \text{ or } 43.3°$

$\cos\beta = \frac{2}{\sqrt{17}} \implies \beta \approx 1.0644 \text{ or } 61.0°$

$\cos\gamma = \frac{-2}{\sqrt{17}} \implies \gamma \approx 2.0772 \text{ or } 119.0°$

36. $\mathbf{u} = \langle -4, 3, 5\rangle \quad \|\mathbf{u}\| = \sqrt{50} = 5\sqrt{2}$

$\cos\alpha = \frac{-4}{5\sqrt{2}} \implies \alpha \approx 2.1721 \text{ or } 124.4°$

$\cos\beta = \frac{3}{5\sqrt{2}} \implies \beta \approx 1.1326 \text{ or } 64.9°$

$\cos\gamma = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}} \implies \gamma \approx \frac{\pi}{4} \text{ or } 45°$

37. $\mathbf{u} = \langle -1, 5, 2\rangle \quad \|\mathbf{u}\| = \sqrt{30}$

$\cos\alpha = \frac{-1}{\sqrt{30}} \implies \alpha \approx 1.7544 \text{ or } 100.5°$

$\cos\beta = \frac{5}{\sqrt{30}} \implies \beta \approx 0.4205 \text{ or } 24.1°$

$\cos\gamma = \frac{2}{\sqrt{30}} \implies \gamma \approx 1.1970 \text{ or } 68.6°$

38. $\mathbf{u} = \langle -2, 6, 1\rangle \quad \|\mathbf{u}\| = \sqrt{41}$

$\cos\alpha = \frac{-2}{\sqrt{41}} \implies \alpha \approx 1.8885 \text{ or } 108.2°$

$\cos\beta = \frac{6}{\sqrt{41}} \implies \alpha \approx 0.3567 \text{ or } 20.4°$

$\cos\gamma = \frac{1}{\sqrt{41}} \implies \alpha \approx 1.4140 \text{ or } 81.0°$

39. $\mathbf{F}_1\text{: } C_1 = \frac{50}{\|\mathbf{F}_1\|} \approx 4.3193$

$\mathbf{F}_2\text{: } C_2 = \frac{80}{\|\mathbf{F}_2\|} \approx 5.4183$

$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$

$\approx 4.3193\langle 10, 5, 3\rangle + 5.4183\langle 12, 7, -5\rangle$

$= \langle 108.2126, 59.5246, -14.1336\rangle$

$\|\mathbf{F}\| \approx 124.310 \text{ lb}$

$\cos\alpha \approx \frac{108.2126}{\|\mathbf{F}\|} \implies \alpha \approx 29.48°$

$\cos\beta \approx \frac{59.5246}{\|\mathbf{F}\|} \implies \beta \approx 61.39°$

$\cos\gamma \approx \frac{-14.1336}{\|\mathbf{F}\|} \implies \gamma \approx 96.53°$

40. $\mathbf{F}_1\text{: } C_1 = \frac{300}{\|\mathbf{F}_1\|} \approx 13.0931$

$\mathbf{F}_2\text{: } C_2 = \frac{100}{\|\mathbf{F}_2\|} \approx 6.3246$

$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$

$\approx 13.0931\langle -20, -10, 5\rangle + 6.3246\langle 5, 15, 0\rangle\rangle$

$= \langle -230.239, -36.062, 65.4655\rangle$

$\|\mathbf{F}\| \approx 242.067 \text{ lb}$

$\cos\alpha \approx \frac{-230.239}{\|\mathbf{F}\|} \implies \alpha \approx 162.02°$

$\cos\beta \approx \frac{-36.062}{\|\mathbf{F}\|} \implies \beta \approx 98.57°$

$\cos\gamma \approx \frac{65.4655}{\|\mathbf{F}\|} \implies \gamma \approx 74.31°$

41. $\overrightarrow{OA} = \langle 0, 10, 10 \rangle$

$$\cos\alpha = \frac{0}{\sqrt{0^2 + 10^2 + 10^2}} = 0 \Rightarrow \alpha = 90°$$

$$\cos\beta = \cos\gamma = \frac{10}{\sqrt{0^2 + 10^2 + 10^2}}$$

$$= \frac{1}{\sqrt{2}} \Rightarrow \beta = \gamma = 45°$$

42. $\mathbf{F}_1 = C_1\langle 0, 10, 10 \rangle$. $\|\mathbf{F}_1\| = 200 = C_1 10\sqrt{2} \Rightarrow C_1 = 10\sqrt{2}$

and $\mathbf{F}_1 = \langle 0, 100\sqrt{2}, 100\sqrt{2} \rangle$

$\mathbf{F}_2 = C_2\langle -4, -6, 10 \rangle$

$\mathbf{F}_3 = C_3\langle 4, -6, 10 \rangle$

$\mathbf{F} = \langle 0, 0, w \rangle$

$\mathbf{F} + \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = \mathbf{0}$

$-4C_2 + 4C_3 = 0 \Rightarrow C_2 = C_3$

$$100\sqrt{2} - 6C_2 - 6C_3 = 0 \Rightarrow C_2 = C_3 = \frac{25\sqrt{2}}{3}N$$

$$W = 10C_2 + 10C_3 + 100\sqrt{2} = \frac{800\sqrt{2}}{3}$$

43. $\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 6, 7 \rangle - \langle 2, 8 \rangle = \langle 4, -1 \rangle$

44. $\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 9, 7 \rangle - \langle 3, 9 \rangle = \langle 6, -2 \rangle$

45. $\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 0, 3, 3 \rangle - \langle -2, 2, 2 \rangle = \langle 2, 1, 1 \rangle$

46. $\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 8, 2, 0 \rangle - \langle 6, 3, -3 \rangle = \langle 2, -1, 3 \rangle$

47. $\mathbf{u} = \langle 2, 3 \rangle, \mathbf{v} = \langle 5, 1 \rangle$

(a) $\mathbf{w}_1 = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \frac{13}{26}\langle 5, 1 \rangle = \left\langle \frac{5}{2}, \frac{1}{2} \right\rangle$

(b) $\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \left\langle -\frac{1}{2}, \frac{5}{2} \right\rangle$

48. $\mathbf{u} = \langle 2, -3 \rangle, \mathbf{v} = \langle 3, 2 \rangle$

(a) $\mathbf{w}_1 = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = 0\mathbf{v} = \langle 0, 0 \rangle$

(b) $\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 2, -3 \rangle$

49. $\mathbf{u} = \langle 2, 1, 2 \rangle, \mathbf{v} = \langle 0, 3, 4 \rangle$

(a) $\mathbf{w}_1 = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v}$

$= \frac{11}{25}\langle 0, 3, 4 \rangle = \left\langle 0, \frac{33}{25}, \frac{44}{25} \right\rangle$

(b) $\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \left\langle 2, -\frac{8}{25}, \frac{6}{25} \right\rangle$

50. $\mathbf{u} = \langle 1, 0, 4 \rangle, \mathbf{v} = \langle 3, 0, 2 \rangle$

(a) $\mathbf{w}_1 = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \frac{11}{13}\langle 3, 0, 2 \rangle = \left\langle \frac{33}{13}, 0, \frac{22}{13} \right\rangle$

(b) $\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 1, 0, 4 \rangle - \left\langle \frac{33}{13}, 0, \frac{22}{13} \right\rangle$

$= \left\langle -\frac{20}{13}, 0, \frac{30}{13} \right\rangle$

51. $\mathbf{u}\cdot\mathbf{v} = \langle u_1, u_2, u_3 \rangle \cdot \langle v_1, v_2, v_3 \rangle = u_1v_1 + u_2v_2 + u_3v_3$

52. The vectors **u** and **v** are orthogonal if $\mathbf{u}\cdot\mathbf{v} = 0$.

The angle θ between **u** and **v** is given by

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|}.$$

53. (a) Orthogonal, $\theta = \frac{\pi}{2}$

(b) Acute, $0 < \theta < \frac{\pi}{2}$

(c) Obtuse, $\frac{\pi}{2} < \theta < \pi$

54. (a) and (b) are defined.

55. See page 784. Direction cosines of $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$ are

$$\cos\alpha = \frac{v_1}{\|\mathbf{v}\|}, \cos\beta = \frac{v_2}{\|\mathbf{v}\|}, \cos\gamma = \frac{v_3}{\|\mathbf{v}\|}.$$

α, β, and γ are the direction angles. See Figure 11.26.

56. See figure 11.29, page 785.

57. (a) $\left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \mathbf{u} \Rightarrow \mathbf{u} = c\mathbf{v} \Rightarrow \mathbf{u}$ and $\mathbf{v}$ are parallel.

(b) $\left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \mathbf{0} \Rightarrow \mathbf{u}\cdot\mathbf{v} = 0 \Rightarrow \mathbf{u}$ and $\mathbf{v}$ are orthogonal.

58. Yes, $\left\|\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v}\right\| = \left\|\frac{\mathbf{v}\cdot\mathbf{u}}{\|\mathbf{u}\|^2}\mathbf{u}\right\|$

$$|\mathbf{u}\cdot\mathbf{v}|\frac{\|\mathbf{v}\|}{\|\mathbf{v}\|^2} = |\mathbf{v}\cdot\mathbf{u}|\frac{\|\mathbf{u}\|}{\|\mathbf{u}\|^2}$$

$$\frac{1}{\|\mathbf{v}\|} = \frac{1}{\|\mathbf{u}\|}$$

$$\|\mathbf{u}\| = \|\mathbf{v}\|$$

59. $\mathbf{u} = \langle 3240, 1450, 2235\rangle$

$\mathbf{v} = \langle 1.35, 2.65, 1.85\rangle$

$\mathbf{u}\cdot\mathbf{v} = 3240(1.35) + 1450(2.65) + 2235(1.85)$

$= \$12{,}351.25$

This represents the total amount that the restaurant earned on its three products.

60. $\mathbf{u} = \langle 3240, 1450, 2235\rangle$

$\mathbf{v} = \langle 1.35, 2.65, 1.85\rangle$

Increase prices by 4%: $1.04\mathbf{v}$

New total amount: $1.04(\mathbf{u}\cdot\mathbf{v}) = 1.04(12{,}351.25)$

$= \$12{,}845.30$

61. Programs will vary.

62. $\|\mathbf{u}\| \approx 9.165$

$\|\mathbf{v}\| \approx 5.745$

$\theta = 90°$

63. Programs will vary.

64. $\left\langle -\frac{21}{26}, \frac{63}{26}, \frac{42}{13}\right\rangle$

65. Because $\mathbf{u}$ appears to be perpendicular to $\mathbf{v}$, the projection of $\mathbf{u}$ onto $\mathbf{v}$ is $\mathbf{0}$. Analytically,

$$\text{proj}_{\mathbf{v}}\mathbf{u} = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v} = \frac{\langle 2, -3\rangle\cdot\langle 6, 4\rangle}{\|\langle 6, 4\rangle\|^2}\langle 6, 4\rangle = 0\langle 6, 4\rangle = \mathbf{0}.$$

66. Because $\mathbf{u}$ appears to be a multiple of $\mathbf{v}$, the projection of $\mathbf{u}$ onto $\mathbf{v}$ is $\mathbf{u}$. Analytically,

$$\text{proj}_{\mathbf{v}}\mathbf{u} = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v} = \frac{\langle -3, -2\rangle\cdot\langle 6, 4\rangle}{\langle 6, 4\rangle\cdot\langle 6, 4\rangle}\langle 6, 4\rangle$$

$$= \frac{-26}{52}\langle 6, 4\rangle = \langle -3, -2\rangle = \mathbf{u}.$$

67. $\mathbf{u} = \frac{1}{2}\mathbf{i} - \frac{2}{3}\mathbf{j}$. Want $\mathbf{u}\cdot\mathbf{v} = 0$.

$\mathbf{v} = 8\mathbf{i} + 6\mathbf{j}$ and $-\mathbf{v} = -8\mathbf{i} - 6\mathbf{j}$ are orthogonal to $\mathbf{u}$.

68. $\mathbf{u} = -8\mathbf{i} + 3\mathbf{j}$. Want $\mathbf{u}\cdot\mathbf{v} = 0$.

$\mathbf{v} = 3\mathbf{i} + 8\mathbf{j}$ and $-\mathbf{v} = -3\mathbf{i} - 8\mathbf{j}$ are orthogonal to $\mathbf{u}$.

69. $\mathbf{u} = \langle 3, 1, -2\rangle$. Want $\mathbf{u}\cdot\mathbf{v} = 0$.

$\mathbf{v} = \langle 0, 2, 1\rangle$ and $-\mathbf{v} = \langle 0, -2, -1\rangle$ are orthogonal to $\mathbf{u}$.

70. $\mathbf{u} = \langle 0, -3, 6\rangle$. Want $\mathbf{u}\cdot\mathbf{v} = 0$.

$\mathbf{v} = \langle 0, 6, 3\rangle$ and $-\mathbf{v} = \langle 0, -6, -3\rangle$ are orthogonal to $\mathbf{u}$.

71. (a) Gravitational Force $\mathbf{F} = -48{,}000\,\mathbf{j}$

$\mathbf{v} = \cos 10°\mathbf{i} + \sin 10°\mathbf{j}$

$$\mathbf{w}_1 = \frac{\mathbf{F}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v} = (\mathbf{F}\cdot\mathbf{v})\mathbf{v} = (-48{,}000)(\sin 10°)\mathbf{v}$$

$$\approx -8335.1(\cos 10°\,\mathbf{i} + \sin 10°\mathbf{j})$$

$\|\mathbf{w}_1\| \approx 8335.1$ lb

(b) $\mathbf{w}_2 = \mathbf{F}\cdot\mathbf{w}_1 = -48{,}000\mathbf{j} + 8335.1(\cos 10°\mathbf{i} + \sin 10°\mathbf{j})$

$= 8208.5\mathbf{i} - 46{,}552.6\mathbf{j}$

$\|\mathbf{w}_2\| \approx 47{,}270.8$ lb

72. $\overrightarrow{OA} = \langle 10, 5, 20\rangle$, $\mathbf{v} = \langle 0, 0, 1\rangle$

$$\text{proj}_{\mathbf{v}}\overrightarrow{OA} = \frac{20}{1^2}\langle 0,0,1\rangle = \langle 0,0,20\rangle$$

$$\|\text{proj}_{\mathbf{v}}\overrightarrow{OA}\| = 20$$

73. $\mathbf{F} = 85\left(\frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j}\right)$

$$\mathbf{v} = 10\mathbf{i}$$

$$W = \mathbf{F}\cdot\mathbf{v} = 425 \text{ ft} \cdot \text{lb}$$

74. $\mathbf{F} = 25(\cos 20°\mathbf{i} + \sin 20°\mathbf{j})$

$$\mathbf{v} = 50\mathbf{i}$$

$$W = \mathbf{F}\cdot\mathbf{v} = 1250\cos 20°$$

$$\approx 1174.6 \text{ ft} \cdot \text{lb}$$

75. False.

Let $\mathbf{u} = \langle 2, 4\rangle$, $\mathbf{v} = \langle 1, 7\rangle$ and $\mathbf{w} = \langle 5, 5\rangle$. Then $\mathbf{u}\cdot\mathbf{v} = 2 + 28 = 30$ and $\mathbf{u}\cdot\mathbf{w} = 10 + 20 = 30$.

76. True

$$\mathbf{w}\cdot(\mathbf{u}+\mathbf{v}) = \mathbf{w}\cdot\mathbf{u} + \mathbf{w}\cdot\mathbf{v}$$

$$= 0 + 0 = 0 \Rightarrow \mathbf{w}$$

and $\mathbf{u} + \mathbf{v}$ are orthogonal.

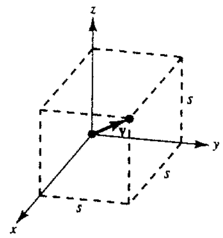
77. Let s = length of a side.

$$\mathbf{v} = \langle s, s, s\rangle$$

$$\|\mathbf{v}\| = s\sqrt{3}$$

$$\cos\alpha = \cos\beta = \cos\gamma = \frac{s}{s\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\alpha = \beta = \gamma = \arccos\left(\frac{1}{\sqrt{3}}\right) \approx 54.7°$$



78. $\mathbf{v}_1 = \langle s, s, s\rangle$

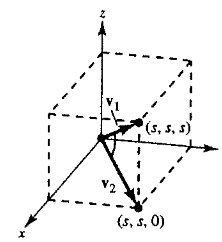
$$\|\mathbf{v}_1\| = s\sqrt{3}$$

$$\mathbf{v}_2 = \langle s, s, 0\rangle$$

$$\|\mathbf{v}_2\| = s\sqrt{2}$$

$$\cos\theta = \frac{s\sqrt{2}}{s\sqrt{3}} = \frac{\sqrt{6}}{3}$$

$$\theta = \arccos\frac{\sqrt{6}}{3} \approx 35.26°$$



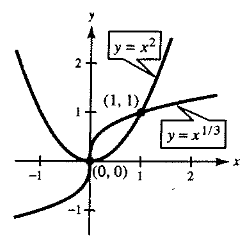
79. (a) The graphs $y_1 = x^2$ and $y_2 = x^{1/3}$ intersect at $(0, 0)$ and $(1, 1)$.

$y'_1 = 2x$ and $y'_2 = \dfrac{1}{3x^{2/3}}$.

At $(0, 0)$, $\pm\langle 1, 0\rangle$ is tangent to y_1 and $\pm\langle 0, 1\rangle$ is tangent to y_2.

At $(1, 1)$, $y'_1 = 2$ and $y'_2 = \dfrac{1}{3}$.

$$\pm\frac{1}{\sqrt{5}}\langle 1, 2\rangle \text{ is tangent to } y_1, \pm\frac{1}{\sqrt{10}}\langle 3, 1\rangle \text{ is tangent to } y_2.$$



(b) At $(0, 0)$, the vectors are perpendicular $(90°)$.

$$\text{At } (1, 1), \cos\theta = \frac{\frac{1}{\sqrt{5}}\langle 1, 2\rangle \cdot \frac{1}{\sqrt{10}}\langle 3, 1\rangle}{(1)(1)} = \frac{5}{\sqrt{50}} = \frac{1}{\sqrt{2}}$$

$$\theta = 45°.$$

80. (a) The graphs $y_1 = x^3$ and $y_2 = x^{1/3}$ intersect at $(-1, -1)$, $(0, 0)$ and $(1, 1)$.

$y'_1 = 3x^2$ and $y'_2 = \dfrac{1}{3x^{2/3}}$.

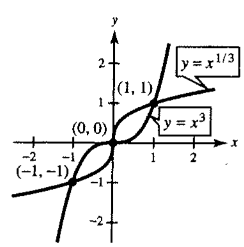
At $(0, 0)$, $\pm\langle 1, 0\rangle$ is tangent to y_1 and $\pm\langle 0, 1\rangle$ is tangent to y_2.

At $(1, 1)$, $y'_1 = 3$ and $y'_2 = \dfrac{1}{3}$.

$$\pm\frac{1}{\sqrt{10}}\langle 1, 3\rangle \text{ is tangent to } y_1, \pm\frac{1}{\sqrt{10}}\langle 3, 1\rangle \text{ is tangent to } y_2.$$

At $(-1, -1)$, $y'_1 = 3$ and $y'_2 = \dfrac{1}{3}$.

$$\pm\frac{1}{\sqrt{10}}\langle 1, 3\rangle \text{ is tangent to } y_1, \pm\frac{1}{\sqrt{10}}\langle 3, 1\rangle \text{ is tangent to } y_2.$$



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80. —CONTINUED—

(b) At (0, 0), the vectors are perpendicular (90°).

$$\text{At } (1, 1), \cos\theta = \frac{\frac{1}{\sqrt{10}}\langle 1, 3\rangle \cdot \frac{1}{\sqrt{10}}\langle 3, 1\rangle}{(1)(1)} = \frac{6}{10} = \frac{3}{5}.$$

$$\theta \approx 0.9273 \text{ or } 53.13°$$

By symmetry, the angle is the same at $(-1, -1)$.

81. (a) The graphs of $y_1 = 1 - x^2$ and $y^2 = x^2 - 1$ intersect at $(1, 0)$ and $(-1, 0)$.

$y'_1 = -2x$ and $y'_2 = 2x$.

At $(1, 0)$, $y'_1 = -2$ and $y'_2 = 2$.

$$\pm\frac{1}{\sqrt{5}}\langle 1, -2\rangle \text{ is tangent to } y_1, \pm\frac{1}{\sqrt{5}}\langle 1, 2\rangle \text{ is tangent to } y_2.$$

At $(-1, 0)$, $y'_1 = 2$ and $y'_2 = -2$.

$$\pm\frac{1}{\sqrt{5}}\langle 1, 2\rangle \text{ is tangent to } y_1, \pm\frac{1}{\sqrt{5}}\langle 1, -2\rangle \text{ is tangent to } y_2.$$

(b) At $(1, 0)$, $\cos\theta = \frac{1}{\sqrt{5}}\langle 1, -2\rangle \cdot \frac{-1}{\sqrt{5}}\langle 1, -2\rangle = \frac{3}{5}$.

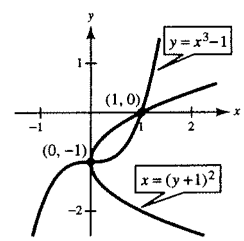
$$\theta \approx 0.9273 \text{ or } 53.13°$$

By symmetry, the angle is the same at $(-1, 0)$.

82. (a) To find the intersection points, rewrite the second equation as $y + 1 = x^3$. Substituting into the first equation

$(y + 1)^2 = x \implies x^6 = x \implies x = 0, 1.$

There are two points of intersection, $(0, -1)$ and $(1, 0)$, as indicated in the figure.



First equation: $(y + 1)^2 = x \implies 2(y + 1)y' = 1 \implies y' = \frac{1}{2(y + 1)}$

At $(1, 0)$, $y' = \frac{1}{2}$.

Second equation: $y = x^3 - 1 \implies y' = 3x^2$. At $(1, 0)$, $y' = 3$.

$$\pm\frac{1}{\sqrt{5}}\langle 2, 1\rangle \text{ unit tangent vectors to first curve, } \pm\frac{1}{\sqrt{10}}\langle 1, 3\rangle \text{ unit tangent vectors to second curve}$$

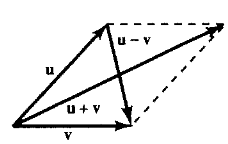
At (0, 1), the unit tangent vectors to the first curve are $\pm\langle 0, 1\rangle$, and the unit tangent vectors to the second curve are $\pm\langle 1, 0\rangle$.

(b) At $(1, 0)$, $\cos\theta = \frac{1}{\sqrt{5}}\langle 2, 1\rangle \cdot \frac{1}{\sqrt{10}}\langle 1, 3\rangle = \frac{5}{\sqrt{50}} = \frac{1}{\sqrt{2}}$.

$$\theta \approx \frac{\pi}{4} \text{ or } 45°$$

At $(0, -1)$ the vectors are perpendicular, $\theta = 90°$.

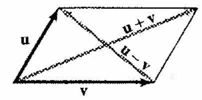
83. In a rhombus, $\|\mathbf{u}\| = \|\mathbf{v}\|$. The diagonals are $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$.



$$\begin{aligned}(\mathbf{u} + \mathbf{v}) \cdot (\mathbf{u} - \mathbf{v}) &= (\mathbf{u} + \mathbf{v}) \cdot \mathbf{u} - (\mathbf{u} + \mathbf{v}) \cdot \mathbf{v} \\ &= \mathbf{u} \cdot \mathbf{u} + \mathbf{v} \cdot \mathbf{u} - \mathbf{u} \cdot \mathbf{v} - \mathbf{v} \cdot \mathbf{v} \\ &= \|\mathbf{u}\|^2 - \|\mathbf{v}\|^2 = 0\end{aligned}$$

Therefore, the diagonals are orthogonal.

84. If **u** and **v** are the sides of the parallelogram, then the diagonals are **u** + **v** and **u** − **v**, as indicated in the figure.



The parallelogram is a rectangle.

$\Leftrightarrow \mathbf{u}\cdot\mathbf{v} = 0$

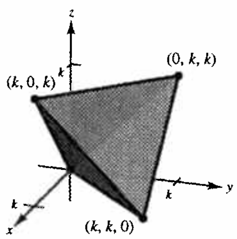
$\Leftrightarrow 2\mathbf{u}\cdot\mathbf{v} = -2\mathbf{u}\cdot\mathbf{v}$

$\Leftrightarrow (\mathbf{u}+\mathbf{v})\cdot(\mathbf{u}+\mathbf{v}) = (\mathbf{u}-\mathbf{v})\cdot(\mathbf{u}-\mathbf{v})$

$\Leftrightarrow \|\mathbf{u}+\mathbf{v}\|^2 = \|\mathbf{u}-\mathbf{v}\|^2$

$\Leftrightarrow$ The diagonals are equal in length.

85. (a)



(b) Length of each edge:

$$\sqrt{k^2+k^2+0^2} = k\sqrt{2}$$

(c) $\cos\theta = \dfrac{k^2}{(k\sqrt{2})(k\sqrt{2})} = \dfrac{1}{2}$

$$\theta = \arccos\left(\frac{1}{2}\right) = 60^\circ$$

(d) $\vec{r_1} = \langle k, k, 0\rangle - \left\langle \frac{k}{2}, \frac{k}{2}, \frac{k}{2}\right\rangle = \left\langle \frac{k}{2}, \frac{k}{2}, -\frac{k}{2}\right\rangle$

$\vec{r_2} = \langle 0, 0, 0\rangle - \left\langle \frac{k}{2}, \frac{k}{2}, \frac{k}{2}\right\rangle = \left\langle -\frac{k}{2}, -\frac{k}{2}, -\frac{k}{2}\right\rangle$

$$\cos\theta = \frac{-\frac{k^2}{4}}{\left(\frac{k}{2}\right)^2\cdot 3} = -\frac{1}{3}$$

$$\theta = 109.5^\circ$$

86. $\mathbf{u} = \langle \cos\alpha, \sin\alpha, 0\rangle$, $\mathbf{v} = \langle \cos\beta, \sin\beta, 0\rangle$

The angle between **u** and **v** is $\alpha - \beta$. (Assuming that $\alpha > \beta$). Also,

$$\cos(\alpha-\beta) = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{\cos\alpha\cos\beta + \sin\alpha\sin\beta}{(1)(1)} = \cos\alpha\cos\beta + \sin\alpha\sin\beta.$$

87.
$$\begin{aligned}\|\mathbf{u}-\mathbf{v}\|^2 &= (\mathbf{u}-\mathbf{v})\cdot(\mathbf{u}-\mathbf{v})\\ &= (\mathbf{u}-\mathbf{v})\cdot\mathbf{u} - (\mathbf{u}-\mathbf{v})\cdot\mathbf{v}\\ &= \mathbf{u}\cdot\mathbf{u} - \mathbf{v}\cdot\mathbf{u} - \mathbf{u}\cdot\mathbf{v} + \mathbf{v}\cdot\mathbf{v}\\ &= \|\mathbf{u}\|^2 - \mathbf{u}\cdot\mathbf{v} - \mathbf{u}\cdot\mathbf{v} + \|\mathbf{v}\|^2\\ &= \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 - 2\mathbf{u}\cdot\mathbf{v}\end{aligned}$$

88.
$$\begin{aligned}\mathbf{u}\cdot\mathbf{v} &= \|\mathbf{u}\|\,\|\mathbf{v}\|\cos\theta\\ |\mathbf{u}\cdot\mathbf{v}| &= \big|\|\mathbf{u}\|\,\|\mathbf{v}\|\cos\theta\big|\\ &= \|\mathbf{u}\|\,\|\mathbf{v}\|\,|\cos\theta|\\ &\le \|\mathbf{u}\|\,\|\mathbf{v}\| \text{ since } |\cos\theta| \le 1.\end{aligned}$$

89.
$$\begin{aligned}\|\mathbf{u}+\mathbf{v}\|^2 &= (\mathbf{u}+\mathbf{v})\cdot(\mathbf{u}+\mathbf{v})\\ &= (\mathbf{u}+\mathbf{v})\cdot\mathbf{u} + (\mathbf{u}+\mathbf{v})\cdot\mathbf{v}\\ &= \mathbf{u}\cdot\mathbf{u} + \mathbf{v}\cdot\mathbf{u} + \mathbf{u}\cdot\mathbf{v} + \mathbf{v}\cdot\mathbf{v}\\ &= \|\mathbf{u}\|^2 + 2\mathbf{u}\cdot\mathbf{v} + \|\mathbf{v}\|^2\\ &\le \|\mathbf{u}\|^2 + 2\|\mathbf{u}\|\,\|\mathbf{v}\| + \|\mathbf{v}\|^2\\ &\le (\|\mathbf{u}\| + \|\mathbf{v}\|)^2\end{aligned}$$

Therefore, $\|\mathbf{u}+\mathbf{v}\| \le \|\mathbf{u}\| + \|\mathbf{v}\|$.

90. Let $\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u}$, as indicated in the figure. Because $\mathbf{w}_1$ is a scalar multiple of $\mathbf{v}$, you can write

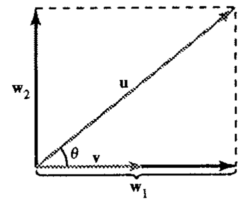
$$\mathbf{u} = \mathbf{w}_1 + \mathbf{w}_2 = c\mathbf{v} + \mathbf{w}_2.$$

Taking the dot product of both sides with $\mathbf{v}$ produces

$$\mathbf{u} \cdot \mathbf{v} = (c\mathbf{v} + \mathbf{w}_2) \cdot \mathbf{v} = c\mathbf{v} \cdot \mathbf{v} + \mathbf{w}_2 \cdot \mathbf{v}$$

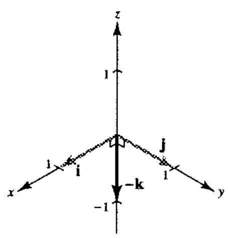
$$= c\|\mathbf{v}\|^2, \text{ since } \mathbf{w}_2 \text{ and } \mathbf{v} \text{ are orthogonol.}$$

Thus, $\mathbf{u} \cdot \mathbf{v} = c\|\mathbf{v}\|^2 \Rightarrow c = \dfrac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}$ and $\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = c\mathbf{v} = \dfrac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v}$.

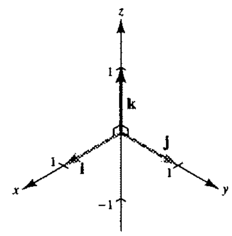


Section 11.4 The Cross Product of Two Vectors in Space

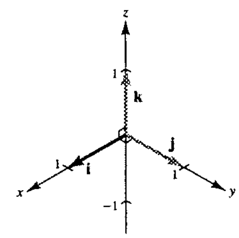
1. $\mathbf{j} \times \mathbf{i} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} = -\mathbf{k}$



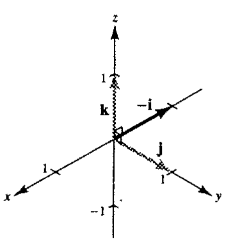
2. $\mathbf{i} \times \mathbf{j} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = \mathbf{k}$



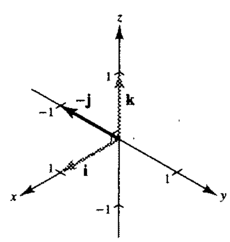
3. $\mathbf{j} \times \mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \mathbf{i}$



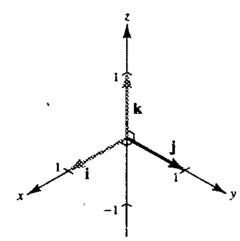
4. $\mathbf{k} \times \mathbf{j} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = -\mathbf{i}$



5. $\mathbf{i} \times \mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} = -\mathbf{j}$



6. $\mathbf{k} \times \mathbf{i} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} = \mathbf{j}$



7. (a) $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 3 & 4 \\ 3 & 7 & 2 \end{vmatrix} = \langle -22, 16, -23 \rangle$

(b) $\mathbf{v} \times \mathbf{u} = -(\mathbf{u} \times \mathbf{v}) = \langle 22, -16, 23 \rangle$

(c) $\mathbf{v} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 7 & 2 \\ 3 & 7 & 2 \end{vmatrix} = \mathbf{0}$

8. (a) $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & 5 \\ 2 & 3 & -2 \end{vmatrix} = \langle -15, 16, 9 \rangle$

(b) $\mathbf{v} \times \mathbf{u} = -(\mathbf{u} \times \mathbf{v}) = \langle 15, -16, -9 \rangle$

(c) $\mathbf{v} \times \mathbf{v} = 0$

9. (a) $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & 3 & 2 \\ 1 & -1 & 5 \end{vmatrix} = \langle 17, -33, -10 \rangle$

(b) $\mathbf{v} \times \mathbf{u} = -(\mathbf{u} \times \mathbf{v}) = \langle -17, 33, 10 \rangle$

(c) $\mathbf{v} \times \mathbf{v} = \mathbf{0}$

10. (a) $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -2 & -2 \\ 1 & 5 & 1 \end{vmatrix} = \langle 8, -5, 17 \rangle$

(b) $\mathbf{v} \times \mathbf{u} = -(\mathbf{u} \times \mathbf{v}) = \langle -8, 5, -17 \rangle$

(c) $\mathbf{v} \times \mathbf{v} = 0$

11. $\mathbf{u} = \langle 2, -3, 1\rangle, \mathbf{v} = \langle 1, -2, 1\rangle$

$$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -3 & 1 \\ 1 & -2 & 1 \end{vmatrix} = -\mathbf{i} - \mathbf{j} - \mathbf{k} = \langle -1, -1, -1\rangle$$

$$\mathbf{u}\cdot(\mathbf{u}\times\mathbf{v}) = 2(-1) + (-3)(-1) + (1)(-1) = 0 \implies \mathbf{u} \perp \mathbf{u}\times\mathbf{v}$$

$$\mathbf{v}\cdot(\mathbf{u}\times\mathbf{v}) = 1(-1) + (-2)(-1) + (1)(-1) = 0 \implies \mathbf{v} \perp \mathbf{u}\times\mathbf{v}$$

12. $\mathbf{u} = \langle -1, 1, 2\rangle, \mathbf{v} = \langle 0, 1, 0\rangle$

$$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 1 & 2 \\ 0 & 1 & 0 \end{vmatrix} = -2\mathbf{i} - \mathbf{k} = \langle -2, 0, -1\rangle$$

$$\mathbf{u}\cdot(\mathbf{u}\times\mathbf{v}) = (-1)(-2) + (1)(0) + (2)(-1)$$
$$= 0 \implies \mathbf{u} \perp \mathbf{u}\times\mathbf{v}$$

$$\mathbf{v}\cdot(\mathbf{u}\times\mathbf{v}) = (0)(-2) + (1)(0) + (0)(-1)$$
$$= 0 \implies \mathbf{v} \perp \mathbf{u}\times\mathbf{v}$$

13. $\mathbf{u} = \langle 12, -3, 0\rangle, \mathbf{v} = \langle -2, 5, 0\rangle$

$$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & -3 & 0 \\ -2 & 5 & 0 \end{vmatrix} = 54\mathbf{k} = \langle 0, 0, 54\rangle$$

$$\mathbf{u}\cdot(\mathbf{u}\times\mathbf{v}) = 12(0) + (-3)(0) + 0(54)$$
$$= 0 \implies \mathbf{u} \perp \mathbf{u}\times\mathbf{v}$$

$$\mathbf{v}\cdot(\mathbf{u}\times\mathbf{v}) = -2(0) + 5(0) + 0(54)$$
$$= 0 \implies \mathbf{v} \perp \mathbf{u}\times\mathbf{v}$$

14. $\mathbf{u} = \langle -10, 0, 6\rangle, \mathbf{v} = \langle 7, 0, 0\rangle$

$$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -10 & 0 & 6 \\ 7 & 0 & 0 \end{vmatrix} = 42\mathbf{j} = \langle 0, 42, 0\rangle$$

$$\mathbf{u}\cdot(\mathbf{u}\times\mathbf{v}) = (-10)(0) + (0)(42) + 6(0)$$
$$= 0 \implies \mathbf{u} \perp \mathbf{u}\times\mathbf{v}$$

$$\mathbf{v}\cdot(\mathbf{u}\times\mathbf{v}) = 7(0) + (0)(42) + (0)(0)$$
$$= 0 \implies \mathbf{v} \perp \mathbf{u}\times\mathbf{v}$$

15. $\mathbf{u} = \mathbf{i} + \mathbf{j} + \mathbf{k}, \mathbf{v} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$

$$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ 2 & 1 & -1 \end{vmatrix} = -2\mathbf{i} + 3\mathbf{j} - \mathbf{k} = \langle -2, 3, -1\rangle$$

$$\mathbf{u}\cdot(\mathbf{u}\times\mathbf{v}) = 1(-2) + 1(3) + 1(-1)$$
$$= 0 \implies \mathbf{u} \perp \mathbf{u}\times\mathbf{v}$$

$$\mathbf{v}\cdot(\mathbf{u}\times\mathbf{v}) = 2(-2) + 1(3) + (-1)(-1)$$
$$= 0 \implies \mathbf{v} \perp \mathbf{u}\times\mathbf{v}$$

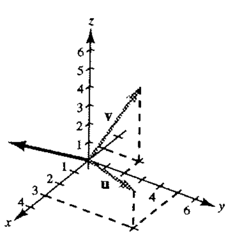
$$(-\mathbf{v})\times\mathbf{u} = -(\mathbf{v}\times\mathbf{u}) = \mathbf{u}\times\mathbf{v}$$

16. $$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 6 & 0 \\ -2 & 1 & 1 \end{vmatrix} = 6\mathbf{i} - \mathbf{j} + 13\mathbf{k}$$

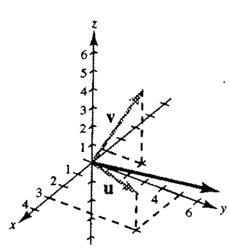
$$\mathbf{u}\cdot(\mathbf{u}\times\mathbf{v}) = 1(6) + 6(-1) = 0 \implies \mathbf{u} \perp (\mathbf{u}\times\mathbf{v})$$

$$\mathbf{v}\cdot(\mathbf{u}\times\mathbf{v}) = -2(6) + 1(-1) + 1(13) = 0 \implies \mathbf{v} \perp (\mathbf{u}\times\mathbf{v})$$

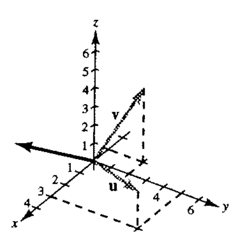
17.



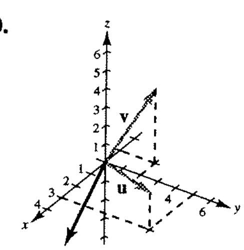
18.



19.



20.



21. $\mathbf{u} = \langle 4, -3.5, 7\rangle$

$\mathbf{v} = \langle -1, 8, 4\rangle$

$$\mathbf{u}\times\mathbf{v} = \left\langle -70, -23, \frac{57}{2}\right\rangle$$

$$\frac{\mathbf{u}\times\mathbf{v}}{\|\mathbf{u}\times\mathbf{v}\|} = \left\langle \frac{-140}{\sqrt{24{,}965}}, \frac{-46}{\sqrt{24{,}965}}, \frac{57}{\sqrt{24{,}965}}\right\rangle$$

22. $\mathbf{u} = \langle -8, -6, 4\rangle$

$\mathbf{v} = \langle 10, -12, -2\rangle$

$\mathbf{u}\times\mathbf{v} = \langle 60, 24, 156\rangle$

$$\frac{\mathbf{u}\times\mathbf{v}}{\|\mathbf{u}\times\mathbf{v}\|} = \frac{1}{36\sqrt{22}}\langle 60, 24, 156\rangle = \left\langle \frac{5}{3\sqrt{22}}, \frac{2}{3\sqrt{22}}, \frac{13}{3\sqrt{22}}\right\rangle$$

23. $\mathbf{u} = -3\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$

$\mathbf{v} = \frac{1}{2}\mathbf{i} - \frac{3}{4}\mathbf{j} + \frac{1}{10}\mathbf{k}$

$\mathbf{u}\times\mathbf{v} = \left\langle -\frac{71}{20}, -\frac{11}{5}, \frac{5}{4}\right\rangle$

$$\frac{\mathbf{u}\times\mathbf{v}}{\|\mathbf{u}\times\mathbf{v}\|} = \frac{20}{\sqrt{7602}}\left\langle -\frac{71}{20}, -\frac{11}{5}, \frac{5}{4}\right\rangle = \left\langle -\frac{71}{\sqrt{7602}}, -\frac{44}{\sqrt{7602}}, \frac{25}{\sqrt{7602}}\right\rangle$$

24. $\mathbf{u} = \frac{2}{3}\mathbf{k}$

$\mathbf{v} = \frac{1}{2}\mathbf{i} + 6\mathbf{k}$

$\mathbf{u}\times\mathbf{v} = \left\langle 0, \frac{1}{3}, 0\right\rangle$

$$\frac{\mathbf{u}\times\mathbf{v}}{\|\mathbf{u}\times\mathbf{v}\|} = \langle 0, 1, 0\rangle$$

25. Programs will vary.

26. $\mathbf{u}\times\mathbf{v} = \langle -50, 40, -34\rangle$

$\|\mathbf{u}\times\mathbf{v}\| \approx 72.498$

27. $\mathbf{u} = \mathbf{j}$

$\mathbf{v} = \mathbf{j} + \mathbf{k}$

$$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \mathbf{i}$$

$A = \|\mathbf{u}\times\mathbf{v}\| = \|\mathbf{i}\| = 1$

28. $\mathbf{u} = \mathbf{i} + \mathbf{j} + \mathbf{k}$

$\mathbf{v} = \mathbf{j} + \mathbf{k}$

$$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} = -\mathbf{j} + \mathbf{k}$$

$A = \|\mathbf{u}\times\mathbf{v}\| = \|-\mathbf{j} + \mathbf{k}\| = \sqrt{2}$

29. $\mathbf{u} = \langle 3, 2, -1\rangle$

$\mathbf{v} = \langle 1, 2, 3\rangle$

$$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 2 & -1 \\ 1 & 2 & 3 \end{vmatrix} = \langle 8, -10, 4\rangle$$

$A = \|\mathbf{u}\times\mathbf{v}\| = \|\langle 8, -10, 4\rangle\| = \sqrt{180} = 6\sqrt{5}$

30. $\mathbf{u} = \langle 2, -1, 0\rangle$

$\mathbf{v} = \langle -1, 2, 0\rangle$

$$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -1 & 0 \\ -1 & 2 & 0 \end{vmatrix} = \langle 0, 0, 3\rangle$$

$A = \|\mathbf{u}\times\mathbf{v}\| = \|\langle 0, 0, 3\rangle\| = 3$

31. $A(1, 1, 1,), B(2, 3, 4), C(6, 5, 2), D(7, 7, 5)$

$\overrightarrow{AB} = \langle 1, 2, 3\rangle, \overrightarrow{AC} = \langle 5, 4, 1\rangle, \overrightarrow{CD} = \langle 1, 2, 3\rangle, \overrightarrow{BD} = \langle 5, 4, 1\rangle$

Since $\overrightarrow{AB} = \overrightarrow{CD}$ and $\overrightarrow{AC} = \overrightarrow{BD}$, the figure is a parallelogram. $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are adjacent sides and

$$\overrightarrow{AB}\times\overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 3 \\ 5 & 4 & 1 \end{vmatrix} = -10\mathbf{i} + 14\mathbf{j} - 6\mathbf{k}.$$

$A = \|\overrightarrow{AB}\times\overrightarrow{AC}\| = \sqrt{332} = 2\sqrt{83}$

32. $A(2, -3, 1), B(6, 5, -1), C(3, -6, 4), D(7, 2, 2)$

$\overrightarrow{AB} = \langle 4, 8, -2\rangle, \overrightarrow{AC} = \langle 1, -3, 3\rangle, \overrightarrow{CD} = \langle 4, 8, -2\rangle, \overrightarrow{BD} = \langle 1, -3, 3\rangle$

Since $\overrightarrow{AB} = \overrightarrow{CD}$ and $\overrightarrow{AC} = \overrightarrow{BD}$, the figure is a parallelogram. $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are adjacent sides and

$$\overrightarrow{AB}\times\overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 8 & -2 \\ 1 & -3 & 3 \end{vmatrix} = \langle 18, -14, -20\rangle.$$

Area $= \|\overrightarrow{AB}\times\overrightarrow{AC}\| = \sqrt{920} = 2\sqrt{230}$

33. $A(0, 0, 0), B(1, 2, 3), C(-3, 0, 0)$

$\overrightarrow{AB} = \langle 1, 2, 3\rangle, \overrightarrow{AC} = \langle -3, 0, 0\rangle$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 3 \\ -3 & 0 & 0 \end{vmatrix} = -9\mathbf{j} + 6\mathbf{k}$$

$$A = \frac{1}{2}\|\overrightarrow{AB} \times \overrightarrow{AC}\| = \frac{1}{2}\sqrt{117} = \frac{3}{2}\sqrt{13}$$

34. $A(2, -3, 4), B(0, 1, 2), C(-1, 2, 0)$

$\overrightarrow{AB} = \langle -2, 4, -2\rangle, \overrightarrow{AC} = \langle -3, 5, -4\rangle$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 4 & -2 \\ -3 & 5 & -4 \end{vmatrix} = -6\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$$

$$A = \frac{1}{2}\|\overrightarrow{AB} \times \overrightarrow{AC}\| = \frac{1}{2}\sqrt{44} = \sqrt{11}$$

35. $A(2, -7, 3), B(-1, 5, 8), C(4, 6, -1)$

$\overrightarrow{AB} = \langle -3, 12, 5\rangle, \overrightarrow{AC} = \langle 2, 13, -4\rangle$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & 12 & 5 \\ 2 & 13 & -4 \end{vmatrix} = \langle -113, -2, -63\rangle$$

$$\text{Area} = \frac{1}{2}\|\overrightarrow{AB} \times \overrightarrow{AC}\| = \frac{1}{2}\sqrt{16{,}742}$$

36. $A(1, 2, 0), B(-2, 1, 0), C(0, 0, 0)$

$\overrightarrow{AB} = \langle -3, -1, 0\rangle, \overrightarrow{AC} = \langle -1, -2, 0\rangle$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & -1 & 0 \\ -1 & -2 & 0 \end{vmatrix} = 5\mathbf{k}$$

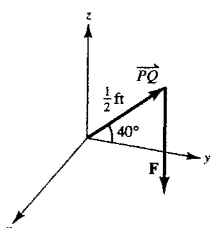
$$A = \frac{1}{2}\|\overrightarrow{AB} \times \overrightarrow{AC}\| = \frac{5}{2}$$

37. $\mathbf{F} = -20\mathbf{k}$

$$\overrightarrow{PQ} = \frac{1}{2}(\cos 40^\circ\mathbf{j} + \sin 40^\circ\mathbf{k})$$

$$\overrightarrow{PQ} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & \cos 40^\circ/2 & \sin 40^\circ/2 \\ 0 & 0 & -20 \end{vmatrix} = -10\cos 40^\circ\mathbf{i}$$

$$\|\overrightarrow{PQ} \times \mathbf{F}\| = 10\cos 40^\circ \approx 7.66 \text{ ft} \cdot \text{lb}$$

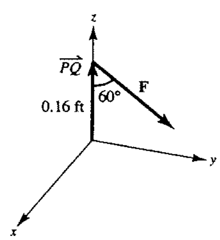


38. $\mathbf{F} = -2000(\cos 30^\circ\mathbf{j} + \sin 30^\circ\mathbf{k}) = -1000\sqrt{3}\mathbf{j} - 1000\mathbf{k}$

$\overrightarrow{PQ} = 0.16\mathbf{k}$

$$\overrightarrow{PQ} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0.16 \\ 0 & -1000\sqrt{3} & -1000 \end{vmatrix} = 160\sqrt{3}\mathbf{i}$$

$$\|\overrightarrow{PQ} \times \mathbf{F}\| = 160\sqrt{3} \text{ ft} \cdot \text{lb}$$



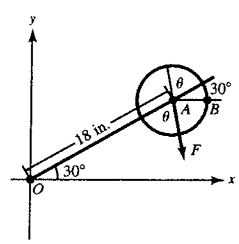
39. (a) Place the wrench in the xy-plane, as indicated in the figure.

The angle from $\overrightarrow{AB}$ to $\mathbf{F}$ is $30^\circ + 180^\circ + \theta = 210^\circ + \theta$.

$\|\overrightarrow{OA}\| = 18$ inches $= 1.5$ feet

$$\overrightarrow{AB} = 1.5[\cos 30^\circ\mathbf{i} + \sin 30^\circ\mathbf{j}] = \frac{3\sqrt{3}}{4}\mathbf{i} + \frac{3}{4}\mathbf{j}$$

$$\mathbf{F} = 60[\cos(210^\circ + \theta)\mathbf{i} + \sin(210^\circ + \theta)\mathbf{j}]$$



$$\overrightarrow{OA} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3\sqrt{3}/4 & 3/4 & 0 \\ 60\cos(210^\circ + \theta) & 60\sin(210^\circ + \theta) & 0 \end{vmatrix}$$

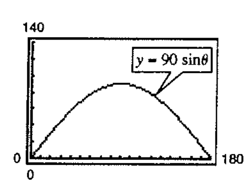
$$= [45\sqrt{3}\sin(210^\circ + \theta) - 45\cos(210^\circ + \theta)]\mathbf{k}$$

$$= [45\sqrt{3}(\sin 210^\circ\cos\theta + \cos 210^\circ\sin\theta) - 45(\cos 210^\circ\cos\theta - \sin 210^\circ\sin\theta)]\mathbf{k}$$

$$= \left[45\sqrt{3}\left(-\frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta\right) - 45\left(-\frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta\right)\right]\mathbf{k}$$

$$= (-90\sin\theta)\mathbf{k}$$

Hence, $\|\overrightarrow{OA} \times \mathbf{F}\| = 90\sin\theta$.



—CONTINUED—

39. —CONTINUED—

(b) When $\theta = 45°$: $\|\overrightarrow{OA} \times \mathbf{F}\| = 90\left(\frac{\sqrt{2}}{2}\right) = 45\sqrt{2} \approx 63.64.$

(c) Let $T = 90 \sin \theta$.

$$\frac{dT}{d\theta} = 90 \cos \theta = 0 \text{ when } \theta = 90°.$$

This is what we expected. When $\theta = 90°$ the pipe wrench is horizontal.

40. (a) B is $-\frac{15}{12} = -\frac{5}{4}$ to the left of A, and one foot upwards:

$$\overrightarrow{AB} = \frac{-5}{4}\mathbf{j} + \mathbf{k}$$

$$\mathbf{F} = -200(\cos \theta\mathbf{j} + \sin \theta\mathbf{k})$$

(b) $$\overrightarrow{AB} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -5/4 & 1 \\ 0 & -200\cos\theta & -200\sin\theta \end{vmatrix} = (250 \sin \theta + 200 \cos \theta)\mathbf{i}$$

$$\|\overrightarrow{AB} \times \mathbf{F}\| = |250 \sin \theta + 200 \cos \theta| = 25|10 \sin \theta + 8 \cos \theta|$$

(c) For $\theta = 30°$,

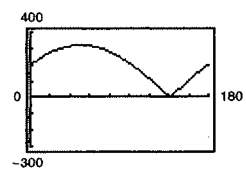
$$\|\overrightarrow{AB} \times \mathbf{F}\| = 25\left(10\left(\frac{1}{2}\right) + 8\left(\frac{\sqrt{3}}{2}\right)\right) = 25(5 + 4\sqrt{3}) \approx 298.2.$$

(d) If $T = \|\overrightarrow{AB} \times \mathbf{F}\|$,

$$\frac{dT}{d\theta} = 25(10 \cos \theta - 8 \sin \theta) = 0 \implies \tan \theta = \frac{5}{4} \implies \theta \approx 51.34°.$$

The vectors are orthogonal.

(e) The zero is $\theta \approx 141.34°$, the angle making $\overrightarrow{AB}$ parallel to $\mathbf{F}$.



41. $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$

42. $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = -1$

43. $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 6$

44. $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} 2 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 2 & 2 \end{vmatrix} = 0$

45. $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 2$

$V = |\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})| = 2$

46. $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} 1 & 3 & 1 \\ 0 & 6 & 6 \\ -4 & 0 & -4 \end{vmatrix} = -72$

$V = |\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})| = 72$

47. $\mathbf{u} = \langle 3, 0, 0 \rangle$

$\mathbf{v} = \langle 0, 5, 1 \rangle$

$\mathbf{w} = \langle 2, 0, 5 \rangle$

$$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} 3 & 0 & 0 \\ 0 & 5 & 1 \\ 2 & 0 & 5 \end{vmatrix} = 75$$

$V = |\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})| = 75$

48. $\mathbf{u} = \langle 1, 1, 0 \rangle$

$\mathbf{v} = \langle 1, 0, 2 \rangle$

$\mathbf{w} = \langle 0, 1, 1 \rangle$

$$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 1 & 1 \end{vmatrix} = -3$$

$V = |\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})| = 3$

49. $\mathbf{u} \times \mathbf{v} = \langle u_1, u_2, u_3 \rangle \cdot \langle v_1, v_2, v_3 \rangle = (u_2v_3 - u_3v_2)\mathbf{i} - (u_1v_3 - u_3v_1)\mathbf{j} + (u_1v_2 - u_2v_1)\mathbf{k}$

50. See Theorem 11.8, page 792.

51. The magnitude of the cross product will increase by a factor of 4.

52. Form the vectors for two sides of the triangle, and compute their cross product:

$$\langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle \times \langle x_3 - x_1, y_3 - y_1, z_3 - z_1 \rangle$$

53. If the vectors are ordered pairs, then the cross product does not exist. False.

54. False, let $\mathbf{u} = \langle 1, 0, 0\rangle$, $\mathbf{v} = \langle 1, 0, 0\rangle$, $\mathbf{w} = \langle -1, 0, 0\rangle$.

Then,

$$\mathbf{u} \times \mathbf{v} = \mathbf{u} \times \mathbf{w} = \mathbf{0}, \text{ but } \mathbf{v} \neq \mathbf{w}.$$

55. True

56. $\mathbf{u} = \langle u_1, u_2, u_3\rangle$, $\mathbf{v} = \langle v_1, v_2, v_3\rangle$, $\mathbf{w} = \langle w_1, w_2, w_3\rangle$

$$\mathbf{u} = u_1\mathbf{i} + u_2\mathbf{j} + u_3\mathbf{k}$$

$$\mathbf{v} \times \mathbf{w} = (v_2w_3 - v_3w_2)\mathbf{i} - (v_1w_3 - v_3w_1)\mathbf{j} + (v_1w_2 - v_2w_1)\mathbf{k}$$

$$\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = u_1(v_2w_3 - v_3w_2) - u_2(v_1w_3 - v_3w_1) + u_3(v_1w_2 - v_2w_1) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

57. $\mathbf{u} = \langle u_1, u_2, u_3\rangle$, $\mathbf{v} = \langle v_1, v_2, v_3\rangle$, $\mathbf{w} = \langle w_1, w_2, w_3\rangle$

$$\mathbf{u} \times (\mathbf{v} + \mathbf{w}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 + w_1 & v_2 + w_2 & v_3 + w_3 \end{vmatrix}$$

$$= [u_2(v_3 + w_3) - u_3(v_2 + w_2)]\mathbf{i} - [u_1(v_3 + w_3) - u_3(v_1 + w_1)]\mathbf{j} + [u_1(v_2 + w_2) - u_2(v_1 + w_1)]\mathbf{k}$$

$$= (u_2v_3 - u_3v_2)\mathbf{i} - (u_1v_3 - u_3v_1)\mathbf{j} + (u_1v_2 - u_2v_1)\mathbf{k} + (u_2w_3 - u_3w_2)\mathbf{i} - (u_1w_3 - u_3w_1)\mathbf{j} + (u_1w_2 - u_2w_1)\mathbf{k}$$

$$= (\mathbf{u} \times \mathbf{v}) + (\mathbf{u} \times \mathbf{w})$$

58. $\mathbf{u} = \langle u_1, u_2, u_3\rangle$, $\mathbf{v} = \langle v_1, v_2, v_3\rangle$, c is a scalar.

$$(c\mathbf{u}) \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ cu_1 & cu_2 & cu_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

$$= (cu_2v_3 - cu_3v_2)\mathbf{i} - (cu_1v_3 - cu_3v_1)\mathbf{j} + (cu_1v_2 - cu_2v_1)\mathbf{k}$$

$$= c[(u_2v_3 - u_3v_2)\mathbf{i} - (u_1v_3 - u_3v_1)\mathbf{j} + (u_1v_2 - u_2v_1)\mathbf{k}] = c(\mathbf{u} \times \mathbf{v})$$

59. $\mathbf{u} = \langle u_1, u_2, u_3\rangle$

$$\mathbf{u} \times \mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ u_1 & u_2 & u_3 \end{vmatrix} = (u_2u_3 - u_3u_2)\mathbf{i} - (u_1u_3 - u_3u_1)\mathbf{j} + (u_1u_2 - u_2u_1)\mathbf{k} = \mathbf{0}$$

60. $$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

$$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} = \mathbf{w} \cdot (\mathbf{u} \times \mathbf{v}) = \begin{vmatrix} w_1 & w_2 & w_3 \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

$$= w_1(u_2v_3 - v_2u_3) - w_2(u_1v_3 - v_1u_3) + w_3(u_1v_2 - v_1u_2)$$

$$= u_1(v_2w_3 - w_2v_3) - u_2(v_1w_3 - w_1v_3) + u_3(v_1w_2 - w_1v_2)$$

$$= \mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$$

61. $\mathbf{u} \times \mathbf{v} = (u_2v_3 - u_3v_2)\mathbf{i} - (u_1v_3 - u_3v_1)\mathbf{j} + (u_1v_2 - u_2v_1)\mathbf{k}$

$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u} = (u_2v_3 - u_3v_2)u_1 + (u_3v_1 - u_1v_3)u_2 + (u_1v_2 - u_2v_1)u_3 = 0$

$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{v} = (u_2v_3 - u_3v_2)v_1 + (u_3v_1 - u_1v_3)v_2 + (u_1v_2 - u_2v_1)v_3 = 0$

Thus, $\mathbf{u} \times \mathbf{v} \perp \mathbf{u}$ and $\mathbf{u} \times \mathbf{v} \perp \mathbf{v}$.

62. If **u** and **v** are scalar multiples of each other, $\mathbf{u} = c\mathbf{v}$ for some scalar c.

$\mathbf{u} \times \mathbf{v} = (c\mathbf{v}) \times \mathbf{v} = c(\mathbf{v} \times \mathbf{v}) = c(\mathbf{0}) = \mathbf{0}$

If $\mathbf{u} \times \mathbf{v} = \mathbf{0}$, then $\|\mathbf{u}\|\,\|\mathbf{v}\| \sin\theta = 0$. (Assume $\mathbf{u} \neq \mathbf{0}$, $\mathbf{v} \neq \mathbf{0}$.) Thus, $\sin\theta = 0$, $\theta = 0$, and **u** and **v** are parallel. Therefore,

$\mathbf{u} = c\mathbf{v}$ for some scalar c.

63. $\|\mathbf{u} \times \mathbf{v}\| = \|\mathbf{u}\|\,\|\mathbf{v}\| \sin\theta$

If **u** and **v** are orthogonal, $\theta = \pi/2$ and $\sin\theta = 1$. Therefore, $\|\mathbf{u} \times \mathbf{v}\| = \|\mathbf{u}\|\,\|\mathbf{v}\|$.

64. $\mathbf{u} = \langle a_1, b_1, c_1\rangle$, $\mathbf{v} = \langle a_2, b_2, c_2\rangle$, $\mathbf{w} = \langle a_3, b_3, c_3\rangle$

$$\mathbf{v} \times \mathbf{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = (b_2c_3 - b_3c_2)\mathbf{i} - (a_2c_3 - a_3c_2)\mathbf{j} + (a_2b_3 - a_3b_2)\mathbf{k}$$

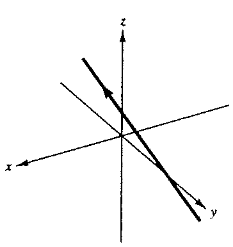
$$\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & b_1 & c_1 \\ (b_2c_3 - b_3c_2) & (a_3c_2 - a_2c_3) & (a_2b_3 - a_3b_2) \end{vmatrix}$$

$$\begin{aligned}\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) &= [b_1(a_2b_3 - a_3b_2) - c_1(a_3c_2 - a_2c_3)]\mathbf{i} - [a_1(a_2b_3 - a_3b_2) - c_1(b_2c_3 - b_3c_2)]\mathbf{j} + \\ &\quad [a_1(a_3c_2 - a_2c_3) - b_1(b_2c_3 - b_3c_2)]\mathbf{k} \\ &= [a_2(a_1a_3 + b_1b_3 + c_1c_3) - a_3(a_1a_2 + b_1b_2 + c_1c_2)]\mathbf{i} + \\ &\quad [b_2(a_1a_3 + b_1b_3 + c_1c_3) - b_3(a_1a_2 + b_1b_2 + c_1c_2)]\mathbf{j} + \\ &\quad [c_2(a_1a_3 + b_1b_3 + c_1c_3) - c_3(a_1a_2 + b_1b_2 + c_1c_2)]\mathbf{k} \\ &= (a_1a_3 + b_1b_3 + c_1c_3)\langle a_2, b_2, c_2\rangle - (a_1a_2 + b_1b_2 + c_1c_2)\langle a_3, b_3, c_3\rangle \\ &= (\mathbf{u} \cdot \mathbf{w})\mathbf{v} - (\mathbf{u} \cdot \mathbf{v})\mathbf{w}\end{aligned}$$

Section 11.5 Lines and Planes in Space

1. $x = 1 + 3t$, $y = 2 - t$, $z = 2 + 5t$

(a)



(b) When $t = 0$ we have $P = (1, 2, 2)$. When $t = 3$ we have $Q = (10, -1, 17)$.

$\overrightarrow{PQ} = \langle 9, -3, 15\rangle$

The components of the vector and the coefficients of t are proportional since the line is parallel to $\overrightarrow{PQ}$.

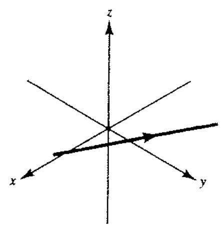
(c) $y = 0$ when $t = 2$. Thus, $x = 7$ and $z = 12$.
Point: $(7, 0, 12)$

$x = 0$ when $t = -\frac{1}{3}$. Point: $\left(0, \frac{7}{3}, \frac{1}{3}\right)$

$z = 0$ when $t = -\frac{2}{5}$. Point: $\left(-\frac{1}{5}, \frac{12}{5}, 0\right)$

2. $x = 2 - 3t, y = 2, z = 1 - t$

(a)



(b) When $t = 0$ we have $P = (2, 2, 1)$. When $t = 2$ we have $Q = (-4, 2, -1)$.

$$\overrightarrow{PQ} = \langle -6, 0, -2 \rangle$$

The components of the vector and the coefficients of t are proportional since the line is parallel to $\overrightarrow{PQ}$.

(c) $z = 0$ when $t = 1$. Thus, $x = -1$ and $y = 2$.

Point: $(-1, 2, 0)$

$x = 0$ when $t = \frac{2}{3}$. Point: $\left(0, 2, \frac{1}{3}\right)$

3. Point: $(0, 0, 0)$

Direction vector: $\mathbf{v} = \langle 1, 2, 3 \rangle$

Direction numbers: 1, 2, 3

(a) Parametric: $x = t, y = 2t, z = 3t$

(b) Symmetric: $x = \frac{y}{2} = \frac{z}{3}$

4. Point: $(0, 0, 0)$

Direction vector: $\mathbf{v} = \left\langle -2, \frac{5}{2}, 1 \right\rangle$

Direction numbers: $-4, 5, 2$

(a) Parametric: $x = -4t, y = 5t, z = 2t$

(b) Symmetric: $\frac{x}{-4} = \frac{y}{5} = \frac{z}{2}$

5. Point: $(-2, 0, 3)$

Direction vector: $\mathbf{v} = \langle 2, 4, -2 \rangle$

Direction numbers: $2, 4, -2$

(a) Parametric: $x = -2 + 2t, y = 4t, z = 3 - 2t$

(b) Symmetric: $\frac{x + 2}{2} = \frac{y}{4} = \frac{z - 3}{-2}$

6. Point: $(-3, 0, 2)$

Direction vector: $\mathbf{v} = \langle 0, 6, 3 \rangle$

Direction numbers: 0, 2, 1

(a) Parametric: $x = -3, y = 2t, z = 2 + t$

(b) Symmetric: $\frac{y}{2} = z - 2, x = -3$

7. Point: $(1, 0, 1)$

Direction vector: $\mathbf{v} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$

Direction numbers: $3, -2, 1$

(a) Parametric: $x = 1 + 3t, y = -2t, z = 1 + t$

(b) Symmetric: $\frac{x - 1}{3} = \frac{y}{-2} = \frac{z - 1}{1}$

8. Point: $(-3, 5, 4)$

Directions numbers: $3, -2, 1$

(a) Parametric: $x = -3 + 3t, y = 5 - 2t, z = 4 + t$

(b) Symmetric: $\frac{x + 3}{3} = \frac{y - 5}{-2} = z - 4$

9. Points: $(5, -3, -2), \left(\frac{-2}{3}, \frac{2}{3}, 1\right)$

Direction vector: $\mathbf{v} = \frac{17}{3}\mathbf{i} - \frac{11}{3}\mathbf{j} - 3\mathbf{k}$

Direction numbers: $17, -11, -9$

(a) Parametric: $x = 5 + 17t, y = -3 - 11t, z = -2 - 9t$

(b) Symmetric: $\frac{x - 5}{17} = \frac{y + 3}{-11} = \frac{z + 2}{-9}$

10. Points: $(2, 0, 2), (1, 4, -3)$

Direction vector: $\langle 1, -4, 5 \rangle$

Direction numbers: $1, -4, 5$

(a) Parametric: $x = 2 + t, y = -4t, z = 2 + 5t$

(b) Symmetric: $x - 2 = \frac{y}{-4} = \frac{z - 2}{5}$

11. Points: $(2, 3, 0), (10, 8, 12)$

Direction vector: $\langle 8, 5, 12\rangle$

Direction numbers: $8, 5, 12$

(a) Parametric: $x = 2 + 8t, y = 3 + 5t, z = 12t$

(b) Symmetric: $\frac{x-2}{8} = \frac{y-3}{5} = \frac{z}{12}$

12. Points: $(0, 0, 25), (10, 10, 0)$

Direction vector: $\langle 10, 10, -25\rangle$

Direction numbers: $2, 2, -5$

(a) Parametric: $x = 2t, y = 2t, z = 25 - 5t$

(b) Symmetric: $\frac{x}{2} = \frac{y}{2} = \frac{z-25}{-5}$

13. Point: $(2, 3, 4)$

Direction vector: $\mathbf{v} = \mathbf{k}$

Direction numbers: $0, 0, 1$

Parametric: $x = 2, y = 3, z = 4 + t$

14. Point: $(-4, 5, 2)$

Direction vector: $\mathbf{v} = \mathbf{j}$

Direction numbers: $0, 1, 0$

Parametric: $x = -4, y = 5 + t, z = 2$

15. Point: $(2, 3, 4)$

Direction vector: $\mathbf{v} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$

Direction numbers: $3, 2, -1$

Parametric: $x = 2 + 3t, y = 3 + 2t, z = 4 - t$

16. Point: $(-4, 5, 2)$

Direction vector: $\mathbf{v} = -\mathbf{i} + 2\mathbf{j} + \mathbf{k}$

Direction numbers: $-1, 2, 1$

Parametric: $x = -4 - t, y = 5 + 2t, z = 2 + t$

17. Point: $(5, -3, -4)$

Direction vector: $\mathbf{v} = \langle 2, -1, 3\rangle$

Direction numbers: $2, -1, 3$

Parametric: $x = 5 + 2t, y = -3 - t, z = -4 + 3t$

18. Point: $(-1, 4, -3)$

Direction vector: $\mathbf{v} = 5\mathbf{i} - \mathbf{j}$

Direction numbers: $5, -1, 0$

Parametric: $x = -1 + 5t, y = 4 - t, z = -3$

19. Point: $(2, 1, 2)$

Direction vector: $\langle -1, 1, 1\rangle$

Direction numbers: $-1, 1, 1$

Parametric: $x = 2 - t, y = 1 + t, z = 2 + t$

20. Point: $(-6, 0, 8)$

Direction vector: $\langle -2, 2, 0\rangle$

Direction numbers: $-2, 2, 0$

Parametric: $x = -6 - 2t, y = 2t, z = 8$

21. Let $t = 0$: $P = (3, -1, -2)$ (other answers possible)

$\mathbf{v} = \langle -1, 2, 0\rangle$ (any nonzero multiple of $\mathbf{v}$ is correct)

22. Let $t = 0$: $P = (0, 5, 4)$ (other answers possible)

$\mathbf{v} = \langle 4, -1, 3\rangle$ (any nonzero multiple of $\mathbf{v}$ is correct)

23. Let each quantity equal 0: $P = (7, -6, -2)$ (other answers possible)

$\mathbf{v} = \langle 4, 2, 1\rangle$ (any nonzero multiple of $\mathbf{v}$ is correct)

24. Let each quantity equal 0: $P = (-3, 0, 3)$ (other answers possible)

$\mathbf{v} = \langle 5, 8, 6\rangle$ (any nonzero multiple of $\mathbf{v}$ is correct)

25.

L_1: $\mathbf{v} = \langle -3, 2, 4\rangle$	$(6, -2, 5)$ on line
L_2: $\mathbf{v} = \langle 6, -4, -8\rangle$	$(6, -2, 5)$ on line
L_3: $\mathbf{v} = \langle -6, 4, 8\rangle$	$(6, -2, 5)$ not on line
L_4: $\mathbf{v} = \langle 6, 4, -6\rangle$	not parallel to L_1, L_2, nor L_3

Hence, L_1 and L_2 are identical.

$L_1 = L_2$ and L_3 are parallel.

26.

L_1: $\mathbf{v} = \langle 4, -2, 3\rangle$	$(8, -5, -9)$ on line
L_2: $\mathbf{v} = \langle 2, 1, 5\rangle$	
L_3: $\mathbf{v} = \langle -8, 4, -6\rangle$	$(8, -5, -9)$ on line
L_4: $\mathbf{v} = \langle -2, 1, 1.5\rangle$	

L_1 and L_2 are identical.

27. At the point of intersection, the coordinates for one line equal the corresponding coordinates for the other line. Thus,

(i) $4t + 2 = 2s + 2$, (ii) $3 = 2s + 3$, and (iii) $-t + 1 = s + 1$.

From (ii), we find that $s = 0$ and consequently, from (iii), $t = 0$. Letting $s = t = 0$, we see that equation (i) is satisfied and therefore the two lines intersect. Substituting zero for s or for t, we obtain the point $(2, 3, 1)$.

$\mathbf{u} = 4\mathbf{i} - \mathbf{k}$ (First line)

$\mathbf{v} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ (Second line)

$$\cos\theta = \frac{|\mathbf{u}\cdot\mathbf{v}|}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{8-1}{\sqrt{17}\sqrt{9}} = \frac{7}{3\sqrt{17}} = \frac{7\sqrt{17}}{51}$$

28. By equating like variables, we have

(i) $-3t + 1 = 3s + 1$, (ii) $4t + 1 = 2s + 4$, and (iii) $2t + 4 = -s + 1$.

From (i) we have $s = -t$, and consequently from (ii), $t = \frac{1}{2}$ and from (iii), $t = -3$. The lines do not intersect.

29. Writing the equations of the lines in parametric form we have

$x = 3t$ $\quad y = 2 - t$ $\quad z = -1 + t$

$x = 1 + 4s$ $\quad y = -2 + s$ $\quad z = -3 - 3s$.

For the coordinates to be equal, $3t = 1 + 4s$ and $2 - t = -2 + s$. Solving this system yields $t = \frac{17}{7}$ and $s = \frac{11}{7}$. When using these values for s and t, the z coordinates are not equal. The lines do not intersect.

30. Writing the equations of the lines in parametric form we have

$x = 2 - 3t$ $\quad y = 2 + 6t$ $\quad z = 3 + t$

$x = 3 + 2s$ $\quad y = -5 + s$ $\quad z = -2 + 4s$.

By equating like variables, we have $2 - 3t = 3 + 2s$, $2 + 6t = -5 + s$, $3 + t = -2 + 4s$. Thus, $t = -1$, $s = 1$ and the point of intersection is $(5, -4, 2)$.

$\mathbf{u} = \langle -3, 6, 1\rangle$ (First line)

$\mathbf{v} = \langle 2, 1, 4\rangle$ (Second line)

$$\cos\theta = \frac{|\mathbf{u}\cdot\mathbf{v}|}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{4}{\sqrt{46}\sqrt{21}} = \frac{4}{\sqrt{966}} = \frac{2\sqrt{966}}{483}$$

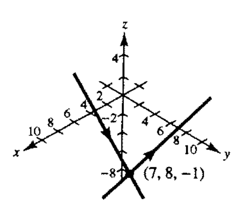
31. $x = 2t + 3$ $\quad x = -2s + 7$

$y = 5t - 2$ $\quad y = s + 8$

$z = -t + 1$ $\quad z = 2s - 1$

Point of intersection: $(7, 8, -1)$

Note: $t = 2$ and $s = 0$

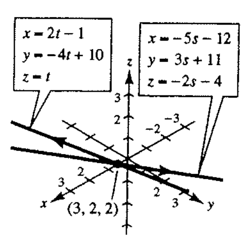


32. $x = 2t - 1$ $\quad x = -5s - 12$

$y = -4t + 10$ $\quad y = 3s + 11$

$z = t$ $\quad z = -2s - 4$

Point of intersection: $(3, 2, 2)$



33. $4x - 3y - 6z = 6$

(a) $P = (0, 0, -1), Q = (0, -2, 0), R = (3, 4, -1)$

$\overrightarrow{PQ} = \langle 0, -2, 1\rangle, \overrightarrow{PR} = \langle 3, 4, 0\rangle$

(b) $\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -2 & 1 \\ 3 & 4 & 0 \end{vmatrix} = \langle -4, 3, 6\rangle$

The components of the cross product are proportional to the coefficients of the variables in the equation. The cross product is parallel to the normal vector.

34. $2x + 3y + 4z = 4$

$P = (0, 0, 1), Q = (2, 0, 0), R = (3, 2, -2)$

(a) $\overrightarrow{PQ} = \langle 2, 0, -1\rangle, \overrightarrow{PR} = \langle 3, 2, -3\rangle$

(b) $\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0 & -1 \\ 3 & 2 & -3 \end{vmatrix} = \langle 2, 3, 4\rangle$

The components of the cross product are proportional (for this choice of P, Q, and R, they are the same) to the coefficients of the variables in the equation. The cross product is parallel to the normal vector.

35. Point: $(2, 1, 2)$

$\mathbf{n} = \mathbf{i} = \langle 1, 0, 0\rangle$

$1(x - 2) + 0(y - 1) + 0(z - 2) = 0$

$x - 2 = 0$

36. Point: $(1, 0, -3)$

$\mathbf{n} = \mathbf{k} = \langle 0, 0, 1\rangle$

$0(x - 1) + 0(y - 0) + 1[z - (-3)] = 0$

$z + 3 = 0$

37. Point: $(3, 2, 2)$

Normal vector: $\mathbf{n} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$

$2(x - 3) + 3(y - 2) - 1(z - 2) = 0$

$2x + 3y - z = 10$

38. Point: $(0, 0, 0)$

Normal vector: $\mathbf{n} = -3\mathbf{i} + 2\mathbf{k}$

$-3(x - 0) + 0(y - 0) + 2(z - 0) = 0$

$-3x + 2z = 0$

39. Point: $(0, 0, 6)$

Normal vector: $\mathbf{n} = -\mathbf{i} + \mathbf{j} - 2\mathbf{k}$

$-1(x - 0) + 1(y - 0) - 2(z - 6) = 0$

$-x + y - 2z + 12 = 0$

$x - y + 2z = 12$

40. Point: $(3, 2, 2)$

Normal vector: $\mathbf{v} = 4\mathbf{i} + \mathbf{j} - 3\mathbf{k}$

$4(x - 3) + (y - 2) - 3(z - 2) = 0$

$4x + y - 3z = 8$

41. Let $\mathbf{u}$ be the vector from $(0, 0, 0)$ to $(1, 2, 3)$:

$\mathbf{u} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$

Let $\mathbf{v}$ be the vector from $(0, 0, 0)$ to $(-2, 3, 3)$:

$\mathbf{v} = -2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$

Normal vector: $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 3 \\ -2 & 3 & 3 \end{vmatrix}$

$= -3\mathbf{i} + (-9)\mathbf{j} + 7\mathbf{k}$

$-3(x - 0) - 9(y - 0) + 7(z - 0) = 0$

$3x + 9y - 7z = 0$

42. Let $\mathbf{u}$ be vector from $(2, 3, -2)$ to $(3, 4, 2)$: $\langle 1, 1, 4\rangle$.

Let $\mathbf{v}$ be vector from $(2, 3, -2)$ to $(1, -1, 0)$: $\langle -1, -4, 2\rangle$.

Normal vector: $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 4 \\ -1 & -4 & 2 \end{vmatrix} = \langle 18, -6, -3\rangle$

$= -3\langle -6, 2, 1\rangle$

$-6(x - 2) + 2(y - 3) + 1(z + 2) = 0$

$-6x + 2y + z = -8$

43. Let $\mathbf{u}$ be the vector from $(1, 2, 3)$ to $(3, 2, 1)$: $\mathbf{u} = 2\mathbf{i} - 2\mathbf{k}$

Let $\mathbf{v}$ be the vector from $(1, 2, 3)$ to $(-1, -2, 2)$: $\mathbf{v} = -2\mathbf{i} - 4\mathbf{j} - \mathbf{k}$

Normal vector: $\left(\frac{1}{2}\mathbf{u}\right) \times (-\mathbf{v}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & -1 \\ 2 & 4 & 1 \end{vmatrix} = 4\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$

$4(x - 1) - 3(y - 2) + 4(z - 3) = 0$

$4x - 3y + 4z = 10$

44. $(1, 2, 3)$, Normal vector: $\mathbf{v} = \mathbf{i}$, $1(x - 1) = 0, x = 1$

45. $(1, 2, 3)$, Normal vector: $\mathbf{v} = \mathbf{k}$, $1(z - 3) = 0, z = 3$

46. The plane passes through the three points $(0, 0, 0)$, $(0, 1, 0)$ $(\sqrt{3}, 0, 1)$.

The vector from $(0, 0, 0)$ to $(0, 1, 0)$: $\mathbf{u} = \mathbf{j}$

The vector from $(0, 0, 0)$ to $(\sqrt{3}, 0, 1)$: $\mathbf{v} = \sqrt{3}\mathbf{i} + \mathbf{k}$

Normal vector: $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 0 \\ \sqrt{3} & 0 & 1 \end{vmatrix} = \mathbf{i} - \sqrt{3}\mathbf{k}$

$x - \sqrt{3}z = 0$

47. The direction vectors for the lines are $\mathbf{u} = -2\mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{v} = -3\mathbf{i} + 4\mathbf{j} - \mathbf{k}$.

Normal vector: $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 1 & 1 \\ -3 & 4 & -1 \end{vmatrix} = -5(\mathbf{i} + \mathbf{j} + \mathbf{k})$

Point of intersection of the lines: $(-1, 5, 1)$

$(x + 1) + (y - 5) + (z - 1) = 0$

$x + y + z = 5$

48. The direction of the line is $\mathbf{u} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$. Choose any point on the line, [(0, 4, 0), for example], and let $\mathbf{v}$ be the vector from $(0, 4, 0)$ to the given point $(2, 2, 1)$:

$\mathbf{v} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$

Normal vector: $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -1 & 1 \\ 2 & -2 & 1 \end{vmatrix} = \mathbf{i} - 2\mathbf{k}$

$(x - 2) - 2(z - 1) = 0$

$x - 2z = 0$

49. Let $\mathbf{v}$ be the vector from $(-1, 1, -1)$ to $(2, 2, 1)$: $\mathbf{v} = 3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$

Let $\mathbf{n}$ be a vector normal to the plane $2x - 3y + z = 3$: $\mathbf{n} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$

Since $\mathbf{v}$ and $\mathbf{n}$ both lie in the plane P, the normal vector to P is

$$\mathbf{v} \times \mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 1 & 2 \\ 2 & -3 & 1 \end{vmatrix} = 7\mathbf{i} + \mathbf{j} - 11\mathbf{k}$$

$7(x - 2) + 1(y - 2) - 11(z - 1) = 0$

$7x + y - 11z = 5$

50. Let $\mathbf{v}$ be the vector from $(3, 2, 1)$ to $(3, 1, -5)$:

$\mathbf{v} = -\mathbf{j} - 6\mathbf{k}$

Let $\mathbf{n}$ be the normal to the given plane: $\mathbf{n} = 6\mathbf{i} + 7\mathbf{j} + 2\mathbf{k}$

Since $\mathbf{v}$ and $\mathbf{n}$ both lie in the plane P, the normal vector to P is:

$$\mathbf{v} \times \mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -1 & -6 \\ 6 & 7 & 2 \end{vmatrix} = 40\mathbf{i} - 36\mathbf{j} + 6\mathbf{k}$$

$$= 2(20\mathbf{i} - 18\mathbf{j} + 3\mathbf{k})$$

$20(x - 3) - 18(y - 2) + 3(z - 1) = 0$

$20x - 18y + 3z = 27$

51. Let $\mathbf{u} = \mathbf{i}$ and let $\mathbf{v}$ be the vector from $(1, -2, -1)$ to $(2, 5, 6)$: $\mathbf{v} = \mathbf{i} + 7\mathbf{j} + 7\mathbf{k}$

Since $\mathbf{u}$ and $\mathbf{v}$ both lie in the plane P, the normal vector to P is:

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 0 \\ 1 & 7 & 7 \end{vmatrix} = -7\mathbf{j} + 7\mathbf{k} = -7(\mathbf{j} - \mathbf{k})$$

$[y - (-2)] - [z - (-1)] = 0$

$y - z = -1$

52. Let $\mathbf{u} = \mathbf{k}$ and let $\mathbf{v}$ be the vector from $(4, 2, 1)$ to $(-3, 5, 7)$: $\mathbf{v} = -7\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$

Since $\mathbf{u}$ and $\mathbf{v}$ both lie in the plane P, the normal vector to P is:

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ -7 & 3 & 6 \end{vmatrix} = -3\mathbf{i} - 7\mathbf{j} = -(3\mathbf{i} + 7\mathbf{j})$$

$3(x - 4) + 7(y - 2) = 0$

$3x + 7y = 26$

53. xy-plane: Let $z = 0$.

Then $0 = 4 - t \Rightarrow t = 4 \Rightarrow x = 1 - 2(4) = -7$ and

$y = -2 + 3(4) = 10$. Intersection: $(-7, 10, 0)$

xz-plane: Let $y = 0$.

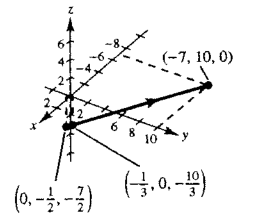
Then $0 = -2 + 3t \Rightarrow t = \frac{2}{3} \Rightarrow x = 1 - 2\left(\frac{2}{3}\right) = -\frac{1}{3}$ and

$z = -4 + \frac{2}{3} = -\frac{10}{3}$. Intersection: $\left(-\frac{1}{3}, 0, -\frac{10}{3}\right)$

yz-plane: Let $x = 0$.

Then $0 = 1 - 2t \Rightarrow t = \frac{1}{2} \Rightarrow y = -2 + 3\left(\frac{1}{2}\right) = -\frac{1}{2}$ and

$z = -4 + \frac{1}{2} = -\frac{7}{2}$. Intersection: $\left(0, -\frac{1}{2}, -\frac{7}{2}\right)$



54. Parametric equations: $x = 2 + 3t, y = -1 + t, z = 3 + 2t$

xy-plane: Let $z = 0$.

Then $3 + 2t = 0 \Rightarrow t = -\frac{3}{2} \Rightarrow x = 2 + 3\left(-\frac{3}{2}\right) = -\frac{5}{2}$ and

$y = -1 + \left(-\frac{3}{2}\right) = -\frac{5}{2}$. Intersection: $\left(-\frac{5}{2}, -\frac{5}{2}, 0\right)$

xz-plane: Let $y = 0$.

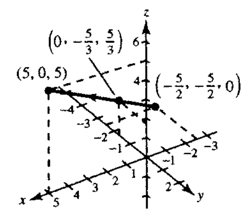
Then $t = 1 \Rightarrow x = 2 + 3(1) = 5$ and

$z = 3 + 2(1) = 5$. Intersection: $(5, 0, 5)$

yz-plane: Let $x = 0$.

Then $2 + 3t = 0 \Rightarrow t = -\frac{2}{3} \Rightarrow y = -1 - \frac{2}{3} = -\frac{5}{3}$ and

$z = 3 + 2\left(-\frac{2}{3}\right) = \frac{5}{3}$. Intersection: $\left(0, -\frac{5}{3}, \frac{5}{3}\right)$



55. Let (x, y, z) be equidistant from $(2, 2, 0)$ and $(0, 2, 2)$.

$$\sqrt{(x-2)^2 + (y-2)^2 + (z-0)^2} = \sqrt{(x-0)^2 + (y-2)^2 + (z-2)^2}$$

$$x^2 - 4x + 4 + y^2 - 4y + 4 + z^2 = x^2 + y^2 - 4y + 4 + z^2 - 4z + 4$$

$$-4x + 8 = -4z + 8$$

$$x - z = 0 \qquad \text{Plane}$$

56. Let (x, y, z) be equidistant from $(-3, 1, 2)$ and $(6, -2, 4)$.

$$\sqrt{(x+3)^2 + (y-1)^2 + (z-2)^2} = \sqrt{(x-6)^2 + (y+2)^2 + (z-4)^2}$$

$$x^2 + 6x + 9 + y^2 - 2y + 1 + z^2 - 4z + 4 = x^2 - 12x + 36 + y^2 + 4y + 4 + z^2 - 8z + 16$$

$$6x - 2y - 4z + 14 = -12x + 4y - 8z + 56$$

$$18x - 6y + 4z - 42 = 0$$

$$9x - 3y + 2z - 21 = 0 \qquad \text{Plane}$$

57. The normal vectors to the planes are

$$\mathbf{n}_1 = \langle 5, -3, 1 \rangle, \mathbf{n}_2 = \langle 1, 4, 7 \rangle, \cos\theta = \frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{\|\mathbf{n}_1\| \|\mathbf{n}_2\|} = 0.$$

Thus, $\theta = \pi/2$ and the planes are orthogonal.

58. The normal vectors to the planes are

$$\mathbf{n}_1 = \langle 3, 1, -4 \rangle, \ \mathbf{n}_2 = \langle -9, -3, 12 \rangle.$$

Since $\mathbf{n}_2 = -3\mathbf{n}_1$, the planes are parallel, but not equal.

59. The normal vectors to the planes are

$$\mathbf{n}_1 = \mathbf{i} - 3\mathbf{j} + 6\mathbf{k},\ \mathbf{n}_2 = 5\mathbf{i} + \mathbf{j} - \mathbf{k},$$

$$\cos\theta = \frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{\|\mathbf{n}_1\|\,\|\mathbf{n}_2\|} = \frac{|5 - 3 - 6|}{\sqrt{46}\sqrt{27}} = \frac{4\sqrt{138}}{414} = \frac{2\sqrt{138}}{207}.$$

Therefore, $\theta = \arccos\left(\frac{2\sqrt{138}}{207}\right) \approx 83.5°$.

60. The normal vectors to the planes are

$$\mathbf{n}_1 = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k},\ \mathbf{n}_2 = \mathbf{i} - 4\mathbf{j} + 2\mathbf{k},$$

$$\cos\theta = \frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{\|\mathbf{n}_1\|\,\|\mathbf{n}_2\|} = \frac{|3 - 8 - 2|}{\sqrt{14}\sqrt{21}} = \frac{1}{\sqrt{6}}.$$

Therefore, $\theta = \arccos\left(\frac{1}{\sqrt{6}}\right) \approx 65.9°$.

61. The normal vectors to the planes are $\mathbf{n}_1 = \langle 1, -5, -1\rangle$ and $\mathbf{n}_2 = \langle 5, -25, -5\rangle$. Since $\mathbf{n}_2 = 5\mathbf{n}_1$, the planes are parallel, but not equal.

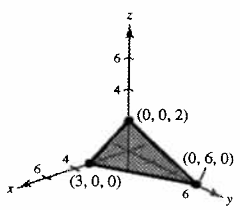
62. The normal vectors to the planes are

$$\mathbf{n}_1 = \langle 2, 0, -1\rangle,\ \mathbf{n}_2 = \langle 4, 1, 8\rangle,$$

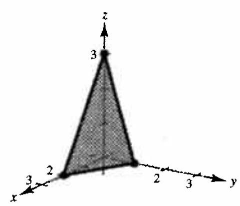
$$\cos\theta = \frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{\|\mathbf{n}_1\|\,\|\mathbf{n}_2\|} = 0.$$

Thus, $\theta = \frac{\pi}{2}$ and the planes are orthogonal.

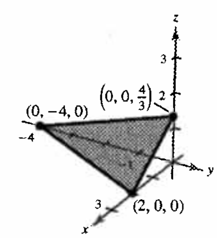
63. $4x + 2y + 6z = 12$



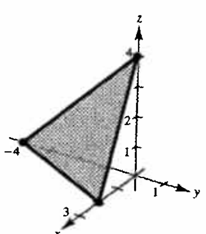
64. $3x + 6y + 2z = 6$



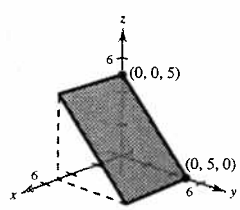
65. $2x - y + 3z = 4$



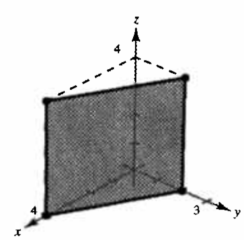
66. $2x - y + z = 4$



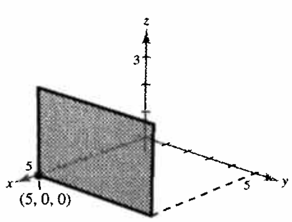
67. $y + z = 5$



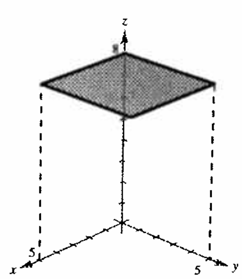
68. $x + 2y = 4$



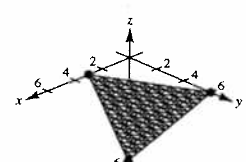
69. $x = 5$



70. $z = 8$



71. $2x + y - z = 6$



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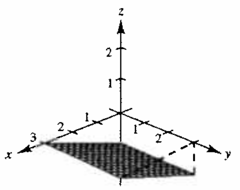
72. $x - 3z = 3$



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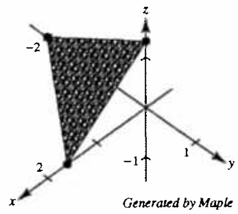
73. $-5x + 4y - 6z + 8 = 0$



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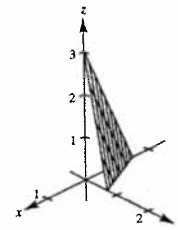
74. $2.1x - 4.7y - z + 3 = 0$



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75. P_1: $\mathbf{n} = \langle 3, -2, 5\rangle$ $\quad (1, -1, 1)$ on plane

P_2: $\mathbf{n} = \langle -6, 4, -10\rangle$ $\quad (1, -1, 1)$ not on plane

P_3: $\mathbf{n} = \langle -3, 2, 5\rangle$

P_4: $\mathbf{n} = \langle 75, -50, 125\rangle$ $\quad (1, -1, 1)$ on plane

P_1 and P_4 are identical.

$P_1 = P_4$ is parallel to P_2.

76. P_1: $\mathbf{n} = \langle -60, 90, 30\rangle$ or $\langle -2, 3, 1\rangle$ $\quad \left(0, 0, \frac{9}{10}\right)$ on plane

P_2: $\mathbf{n} = \langle 6, -9, -3\rangle$ or $\langle -2, 3, 1\rangle$ $\quad \left(0, 0, -\frac{2}{3}\right)$ on plane

P_3: $\mathbf{n} = \langle -20, 30, 10\rangle$ or $\langle -2, 3, 1\rangle$ $\quad \left(0, 0, \frac{5}{6}\right)$ on plane

P_4: $\mathbf{n} = \langle 12, -18, 6\rangle$ or $\langle -2, 3, -1\rangle$

P_1, P_2, and P_3 are parallel.

77. Each plane passes through the points

$(c, 0, 0)$, $(0, c, 0)$, and $(0, 0, c)$.

78. $x + y = c$

Each plane is parallel to the z-axis.

79. If $c = 0$, $z = 0$ is xy-plane.

If $c \neq 0$, $cy + z = 0 \implies y = \dfrac{-1}{c}z$ is a plane parallel to x-axis and passing through the points $(0, 0, 0)$ and $(0, 1, -c)$.

80. $x + cz = 0$

If $c = 0$, $x = 0$ is the yz-plane.

If $c \neq 0$, $x + cz = 0$ is a plane parallel to the y-axis.

81. The normals to the planes are $\mathbf{n}_1 = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{n}_2 = \mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$. The direction vector for the line is

$$\mathbf{n}_2 \times \mathbf{n}_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -4 & 2 \\ 3 & 2 & -1 \end{vmatrix} = 7(\mathbf{j} + 2\mathbf{k}).$$

Now find a point of intersection of the planes.

$$\begin{aligned} 6x + 4y - 2z &= 14 \\ x - 4y + 2z &= 0 \\ 7x \qquad\qquad &= 14 \\ x &= 2 \end{aligned}$$

Substituting 2 for x in the second equation, we have $-4y + 2z = -2$ or $z = 2y - 1$. Letting $y = 1$, a point of intersection is $(2, 1, 1)$.

$x = 2, y = 1 + t, z = 1 + 2t$

82. The normals to the planes are $\mathbf{n}_1 = \langle 6, -3, 1\rangle$ and $\mathbf{n}_2 = \langle -1, 1, 5\rangle$.

The direction vector for the line is

$$\mathbf{n}_1 \times \mathbf{n}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & -3 & 1 \\ -1 & 1 & 5 \end{vmatrix} = \langle -16, -31, 3\rangle.$$

Now find a point of intersection of the planes.

$$\begin{aligned} 6x - 3y + z &= 5 \implies & 6x - 3y + z &= 5 \\ -x + y + 5z &= 5 \implies & -6x + 6y + 30z &= 30 \\ & & 3y + 31z &= 35 \end{aligned}$$

Let $y = -9, z = 2 \implies x = -4 \implies (-4, -9, 2)$.

$x = -4 - 16t, y = -9 - 31t, z = 2 + 3t$

83. Writing the equation of the line in parametric form and substituting into the equation of the plane we have:

$$x = \frac{1}{2} + t,\ y = \frac{-3}{2} - t,\ z = -1 + 2t$$

$$2\left(\frac{1}{2} + t\right) - 2\left(\frac{-3}{2} - t\right) + (-1 + 2t) = 12,\ t = \frac{3}{2}$$

Substituting $t = 3/2$ into the parametric equations for the line we have the point of intersection $(2, -3, 2)$. The line does not lie in the plane.

84. Writing the equation of the line in parametric form and substituting into the equation of the plane we have:

$$x = 1 + 4t,\ y = 2t,\ z = 3 + 6t$$

$$2(1 + 4t) + 3(2t) = -5,\ t = \frac{-1}{2}$$

Substituting $t = -\frac{1}{2}$ into the parametric equations for the line we have the point of intersection $(-1, -1, 0)$. The line does not lie in the plane.

85. Writing the equation of the line in parametric form and substituting into the equation of the plane we have:

$$x = 1 + 3t,\ y = -1 - 2t,\ z = 3 + t$$

$$2(1 + 3t) + 3(-1 - 2t) = 10,\ -1 = 10, \text{ contradiction}$$

Therefore, the line does not intersect the plane.

86. Writing the equation of the line in parametric form and substituting into the equation of the plane we have:

$$x = 4 + 2t,\ y = -1 - 3t,\ z = -2 + 5t$$

$$5(4 + 2t) + 3(-1 - 3t) = 17,\ t = 0$$

Substituting $t = 0$ into the parametric equations for the line we have the point of intersection $(4, -1, -2)$. The line does not lie in the plane.

87. Point: $Q(0, 0, 0)$

Plane: $2x + 3y + z - 12 = 0$

Normal to plane: $\mathbf{n} = \langle 2, 3, 1\rangle$

Point in plane: $P(6, 0, 0)$

Vector $\overrightarrow{PQ} = \langle -6, 0\ 0\rangle$

$$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \frac{|-12|}{\sqrt{14}} = \frac{6\sqrt{14}}{7}$$

88. Point: $Q(0, 0, 0)$

Plane: $8x - 4y + z = 8$

Normal to plane: $\mathbf{n} = \langle 8, -4, 1\rangle$

Point in plane: $P\langle 1, 0, 0\rangle$

Vector: $\overrightarrow{PQ} = \langle -1, 0, 0\rangle$

$$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \frac{|-8|}{\sqrt{81}} = \frac{8}{9}$$

89. Point: $Q(2, 8, 4)$

Plane: $2x + y + z = 5$

Normal to plane: $\mathbf{n} = \langle 2, 1, 1\rangle$

Point in plane: $P\langle 0, 0, 5\rangle$

Vector: $\overrightarrow{PQ} = \langle 2, 8, -1\rangle$

$$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \frac{11}{\sqrt{6}} = \frac{11\sqrt{6}}{6}$$

90. Point: $Q(3, 2, 1)$

Plane: $x - y + 2z = 4$

Normal to plane: $\mathbf{n} = \langle 1, -1, 2\rangle$

Point in plane: $P\langle 4, 0, 0\rangle$

Vector: $\overrightarrow{PQ} = \langle -1, 2, 1\rangle$

$$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \frac{|-1|}{\sqrt{6}} = \frac{1}{\sqrt{6}} = \frac{\sqrt{6}}{6}$$

91. The normal vectors to the planes are $\mathbf{n}_1 = \langle 1, -3, 4\rangle$ and $\mathbf{n}_2 = \langle 1, -3, 4\rangle$. Since $\mathbf{n}_1 = \mathbf{n}_2$, the planes are parallel. Choose a point in each plane.

$P = (10, 0, 0)$ is a point in $x - 3y + 4z = 10$.

$Q = (6, 0, 0)$ is a point in $x - 3y + 4z = 6$.

$$\overrightarrow{PQ} = \langle -4, 0, 0\rangle,\ D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}_1|}{\|\mathbf{n}_1\|} = \frac{4}{\sqrt{26}} = \frac{2\sqrt{26}}{13}$$

92. The normal vectors to the planes are $\mathbf{n}_1 = \langle 4, -4, 9\rangle$ and $\mathbf{n}_2 = \langle 4, -4, 9\rangle$. Since $\mathbf{n}_1 = \mathbf{n}_2$, the planes are parallel. Choose a point in each plane.

$P = (-5, 0, 3)$ is a point in $4x - 4y + 9z = 7$.

$Q = (0, 0, 2)$ is a point in $4x - 4y + 9z = 18$.

$\overrightarrow{PQ} = \langle 5, 0, -1\rangle$

$$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}_1|}{\|\mathbf{n}_1\|} = \frac{11}{\sqrt{113}} = \frac{11\sqrt{113}}{113}$$

93. The normal vectors to the planes are $\mathbf{n}_1 = \langle -3, 6, 7\rangle$ and $\mathbf{n}_2 = \langle 6, -12, -14\rangle$. Since $\mathbf{n}_2 = -2\mathbf{n}_1$, the planes are parallel. Choose a point in each plane.

$P = (0, -1, 1)$ is a point in $-3x + 6y + 7z = 1$.

$Q = \left(\frac{25}{6}, 0, 0\right)$ is a point in $6x - 12y - 14z = 25$.

$$\overrightarrow{PQ} = \left\langle \frac{25}{6}, 1, -1\right\rangle$$

$$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}_1|}{\|\mathbf{n}_1\|} = \frac{|-27/2|}{\sqrt{94}} = \frac{27}{2\sqrt{94}} = \frac{27\sqrt{94}}{188}$$

94. The normal vectors to the planes are $\mathbf{n}_1 = \langle 2, 0, -4\rangle$ and $\mathbf{n}_2 = \langle 2, 0, -4\rangle$. Since $\mathbf{n}_1 = \mathbf{n}_2$, the planes are parallel. Choose a point in each plane.

$P = (2, 0, 0)$ is a point in $2x - 4z = 4$.
$Q = (5, 0, 0)$ is a point in $2x - 4z = 10$.

$$\overrightarrow{PQ} = \langle 3, 0, 0\rangle, D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}_1|}{\|\mathbf{n}_1\|} = \frac{6}{\sqrt{20}} = \frac{3\sqrt{5}}{5}$$

95. $\mathbf{u} = \langle 4, 0, -1\rangle$ is the direction vector for the line. $Q(1, 5, -2)$ is the given point, and $P(-2, 3, 1)$ is on the line. Hence, $\overrightarrow{PQ} = \langle 3, 2, -3\rangle$ and

$$\overrightarrow{PQ} \times \mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 2 & -3 \\ 4 & 0 & -1 \end{vmatrix} = \langle -2, -9, -8\rangle.$$

$$D = \frac{\|\overrightarrow{PQ} \times \mathbf{u}\|}{\|\mathbf{u}\|} = \frac{\sqrt{149}}{\sqrt{17}} = \frac{\sqrt{2533}}{17}$$

96. $\mathbf{u} = \langle 2, 1, 2\rangle$ is the direction vector for the line.

$P = (0, -3, 2)$ is a point on the line (let $t = 0$).

$\overrightarrow{PQ} = \langle 1, 1, 2\rangle$

$$\overrightarrow{PQ} \times \mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 2 \\ 2 & 1 & 2 \end{vmatrix} = \langle 0, 2, -1\rangle$$

$$D = \frac{\|\overrightarrow{PQ} \times \mathbf{u}\|}{\|\mathbf{u}\|} = \frac{\sqrt{5}}{\sqrt{9}} = \frac{\sqrt{5}}{3}$$

97. $\mathbf{u} = \langle -1, 1, -2\rangle$ is the direction vector for the line.

$Q = (-2, 1, 3)$ is the given point, and $P = (1, 2, 0)$ is on the line (let $t = 0$ in the parametric equations for the line).

Hence, $\overrightarrow{PQ} = \langle -3, -1, 3\rangle$ and

$$\overrightarrow{PQ} \times \mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & -1 & 3 \\ -1 & 1 & -2 \end{vmatrix} = \langle -1, -9, -4\rangle.$$

$$D = \frac{\|\overrightarrow{PQ} \times \mathbf{u}\|}{\|\mathbf{u}\|} = \frac{\sqrt{1 + 81 + 16}}{\sqrt{1 + 1 + 4}} = \frac{\sqrt{98}}{6} = \frac{7}{\sqrt{3}} = \frac{7\sqrt{3}}{3}$$

98. $\mathbf{u} = \langle 0, 3, 1\rangle$ is the direction vector for the line.

$Q = (4, -1, 5)$ is the given point, and $P = (3, 1, 1)$ is on the line. Hence, $\overrightarrow{PQ} = \langle 1, -2, 4\rangle$ and

$$\overrightarrow{PQ} \times \mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 4 \\ 0 & 3 & 1 \end{vmatrix} = \langle -14, -1, 3\rangle.$$

$$D = \frac{\|\overrightarrow{PQ} \times \mathbf{u}\|}{\|\mathbf{u}\|} = \frac{\sqrt{14^2 + 1 + 9}}{\sqrt{9 + 1}} = \sqrt{\frac{206}{10}}$$
$$= \sqrt{\frac{103}{5}} = \frac{\sqrt{515}}{5}$$

99. The direction vector for L_1 is $\mathbf{v}_1 = \langle -1, 2, 1\rangle$.

The direction vector for L_2 is $\mathbf{v}_2 = \langle 3, -6, -3\rangle$.

Since $\mathbf{v}_2 = -3\mathbf{v}_1$, the lines are parallel.

Let $Q = (2, 3, 4)$ to be a point on L_1 and $P = (0, 1, 4)$ a point on L_2. $\overrightarrow{PQ} = \langle 2, 0, 0\rangle$.
$\mathbf{u} = \mathbf{v}_2$ is the direction vector for L_2.

$$\overrightarrow{PQ} \times \mathbf{v}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 2 & 0 \\ 3 & -6 & -3 \end{vmatrix} = \langle -6, 6, -18\rangle$$

$$D = \frac{\|\overrightarrow{PQ} \times \mathbf{v}_2\|}{\|\mathbf{v}_2\|} = \frac{\sqrt{36 + 36 + 324}}{\sqrt{9 + 36 + 9}} = \sqrt{\frac{396}{54}}$$
$$= \sqrt{\frac{22}{3}} = \frac{\sqrt{66}}{3}$$

100. The direction vector for L_1 is $\mathbf{v}_1 = \langle 6, 9, -12\rangle$.

The direction vector for L_2 is $\mathbf{v}_2 = \langle 4, 6, -8\rangle$.

Since $\mathbf{v}_1 = \frac{3}{2}\mathbf{v}_2$, the lines are parallel.

Let $Q = (3, -2, 1)$ to be a point on L_1 and $P = (-1, 3, 0)$ a point on L_2. $\overrightarrow{PQ} = \langle 4, -5, 1\rangle$.
$\mathbf{u} = \mathbf{v}_2$ is the direction vector for L_2.

$$\overrightarrow{PQ} \times \mathbf{v}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -5 & 1 \\ 4 & 6 & -8 \end{vmatrix} = \langle 34, 36, 44\rangle$$

$$D = \frac{\|\overrightarrow{PQ} \times \mathbf{v}_2\|}{\|\mathbf{v}_2\|} = \frac{\sqrt{34^2 + 36^2 + 44^2}}{\sqrt{16 + 36 + 64}} = \frac{\sqrt{4388}}{\sqrt{116}}$$
$$= \sqrt{\frac{1097}{29}} = \frac{\sqrt{31813}}{29}$$

101. The parametric equations of a line L parallel to $\mathbf{v} = \langle a, b, c,\rangle$ and passing through the point $P(x_1, y_1, z_1)$ are

$$x = x_1 + at, y = y_1 + bt, z = z_1 + ct.$$

The symmetric equations are

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}.$$

102. The equation of the plane containing $P(x_1, y_1, z_1)$ and having normal vector $\mathbf{n} = \langle a, b, c\rangle$ is

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0.$$

You need $\mathbf{n}$ and P to find the equation.

103. Solve the two linear equations representing the planes to find two points of intersection. Then find the line determined by the two points.

104. $x = a$: plane parallel to yz-plane containing $(a, 0, 0)$

$y = b$: plane parallel to xz-plane containing $(0, b, 0)$

$z = c$: plane parallel to xy-plane containing $(0, 0, c)$

105. (a) The planes are parallel if their normal vectors are parallel:

$$\langle a_1, b_1, c_1\rangle = t\langle a_2, b_2, c_2\rangle,\ t \neq 0$$

(b) The planes are perpendicular if their normal vectors are perpendicular:

$$\langle a_1, b_1, c_1\rangle \cdot \langle a_2, b_2, c_2\rangle = 0$$

106. Yes. If $\mathbf{v}_1$ and $\mathbf{v}_2$ are the direction vectors for the lines L_1 and L_2, then $\mathbf{v} = \mathbf{v}_1 \times \mathbf{v}_2$ is perpendicular to both L_1 and L_2.

107. An equation for the plane is

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.$$

For example, letting $y = z = 0$, the x-intercept is $(a, 0, 0)$.

108. (a) Sphere

$$(x - 3)^2 + (y + 2)^2 + (z - 5)^2 = 16$$
$$x^2 + y^2 + z^2 - 6x + 4y - 10z + 22 = 0$$

(b) Parallel planes

$$4x - 3y + z = 10 \pm 4\|\mathbf{n}\| = 10 \pm 4\sqrt{26}$$

109. $0.04x - 0.64y + z = 3.4 \implies z = 3.4 - 0.04x + 0.64y$

(a)

Year	1994	1995	1996	1997	1998	1999	2000
x	5.8	6.2	6.4	6.6	6.5	6.3	6.1
y	8.7	8.2	8.0	7.7	7.4	7.3	7.1
z	8.8	8.4	8.4	8.2	7.8	7.9	7.8
Model, z'	8.74	8.40	8.26	8.06	7.88	7.82	7.70

The approximations are close to the actual values.

(b) According to the model, if x and z decrease, then so will y, the consumption of reduced-fat milk.

110. On one side we have the points $(0, 0, 0)$, $(6, 0, 0)$, and $(-1, -1, 8)$.

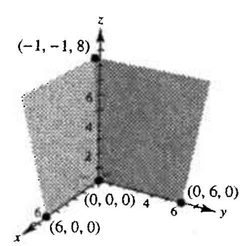
$$\mathbf{n}_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 0 & 0 \\ -1 & -1 & 8 \end{vmatrix} = -48\mathbf{j} - 6\mathbf{k}$$

On the adjacent side we have the points $(0, 0, 0)$, $(0, 6, 0)$, and $(-1, -1, 8)$.

$$\mathbf{n}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 6 & 0 \\ -1 & -1 & 8 \end{vmatrix} = 48\mathbf{i} + 6\mathbf{k}$$

$$\cos\theta = \frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{\|\mathbf{n}_1\|\,\|\mathbf{n}_2\|} = \frac{36}{2340} = \frac{1}{65}$$

$$\theta = \arccos\frac{1}{65} \approx 89.1°$$



111. L_1: $x_1 = 6 + t, y_1 = 8 - t, z_1 = 3 + t$

L_2: $x_2 = 1 + t, y_2 = 2 + t, z_2 = 2t$

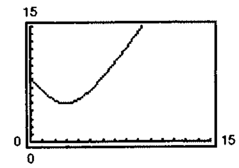
(a) At $t = 0$, the first insect is at $P_1 = (6, 8, 3)$ and the second insect is at $P_2 = (1, 2, 0)$.

Distance $= \sqrt{(6-1)^2 + (8-2)^2 + (3-0)^2} = \sqrt{70} \approx 8.37$ inches

(b) Distance $= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

$= \sqrt{5^2 + (6-2t)^2 + (3-t)^2}$

$= \sqrt{5t^2 - 30t + 70},\ 0 \le t \le 10$



(c) The distance is never zero.

(d) Using a graphing utility, the minimum distance is 5 inches when $t = 3$ minutes.

112. First find the distance D from the point $Q = (-3, 2, 4)$ to the plane. Let $P = (4, 0, 0)$ be on the plane. $\mathbf{n} = \langle 2, 4, -3\rangle$ is the normal to the plane.

$$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \frac{|\langle -7, 2, 4\rangle \cdot \langle 2, 4, -3\rangle|}{\sqrt{4 + 16 + 9}} = \frac{|-14 + 8 - 12|}{\sqrt{29}} = \frac{18}{\sqrt{29}} = \frac{18\sqrt{29}}{29}$$

The equation of the sphere with center $(-3, 2, 4)$ and radius $18\sqrt{29}/29$ is $(x+3)^2 + (y-2)^2 + (z-4)^2 = \frac{324}{29}$.

113. The direction vector $\mathbf{v}$ of the line is the normal to the plane, $\mathbf{v} = \langle 3, -1, 4\rangle$.

The parametric equations of the line are $x = 5 + 3t, y = 4 - t, z = -3 + 4t$.

To find the point of intersection, solve for t in the following equation:

$$3(5 + 3t) - (4 - t) + 4(-3 + 4t) = 7$$
$$26t = 8$$
$$t = \frac{4}{13}$$

Point of intersection is $\left(5 + 3\left(\frac{4}{13}\right), 4 - \frac{4}{13}, -3 + 4\left(\frac{4}{13}\right)\right) = \left(\frac{77}{13}, \frac{48}{13}, -\frac{23}{13}\right)$

114. The normal to the plane, $\mathbf{n} = \langle 2, -1, -3\rangle$ is perpendicular to the direction vector $\mathbf{v} = \langle 2, 4, 0\rangle$ of the line because $\langle 2, -1, -3\rangle \cdot \langle 2, 4, 0\rangle = 0$.

Hence, the plane is parallel to the line. To find the distance between them, let $Q = (-2, -1, 4)$ be on the line and $P = (2, 0, 0)$ on the plane. $\overrightarrow{PQ} = \langle -4, -1, 4\rangle$.

$$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \frac{|\langle -4, -1, 4\rangle \cdot \langle 2, -1, -3\rangle|}{\sqrt{4 + 1 + 9}} = \frac{19}{\sqrt{14}} = \frac{19\sqrt{14}}{14}$$

115. The direction vector of the line L through $(1, -3, 1)$ and $(3, -4, 2)$ is $\mathbf{v} = \langle 2, -1, 1\rangle$.

The parametric equations for L are $x = 1 + 2t, y = -3 - t, z = 1 + t$.

Substituting these equations into the equation of the plane gives

$$(1 + 2t) - (-3 - t) + (1 + t) = 2$$
$$4t = -3$$
$$t = -\frac{3}{4}.$$

Point of intersection is $\left(1 + 2\left(-\frac{3}{4}\right), -3 + \frac{3}{4}, 1 - \frac{3}{4}\right) = \left(-\frac{1}{2}, -\frac{9}{4}, \frac{1}{4}\right)$

116. The unknown line L is perpendicular to the normal vector $\mathbf{n} = \langle 1, 1, 1\rangle$ of the plane, and perpendicular to the direction vector $\mathbf{u} = \langle 1, 1, -1\rangle$. Hence, the direction vector of L is

$$\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = \langle -2, 2, 0\rangle.$$

The parametric equations for L are $x = 1 - 2t, y = 2t, z = 2$.

117. True

118. False. They may be skew lines. (See Section Project)

119. True

120. False. The lines $x = t, y = 0, z = 1$ and $x = 0, y = t, z = 1$ are both parallel to the plane $z = 0$, but the lines are not parallel.

Section 11.6 Surfaces in Space

1. Ellipsoid
Matches graph (c)

2. Hyperboloid of two sheets
Matches graph (e)

3. Hyperboloid of one sheet
Matches graph (f)

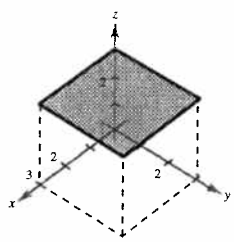
4. Elliptic cone
Matches graph (b)

5. Elliptic paraboloid
Matches graph (d)

6. Hyperbolic paraboloid
Matches graph (a)

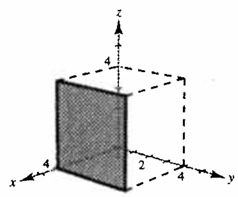
7. $z = 3$

Plane parallel to the xy-coordinate plane



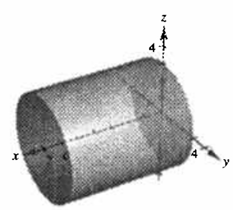
8. $x = 4$

Plane parallel to the yz-coordinate plane



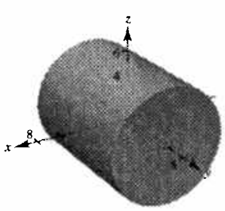
9. $y^2 + z^2 = 9$

The x-coordinate is missing so we have a cylindrical surface with rulings parallel to the x-axis. The generating curve is a circle.



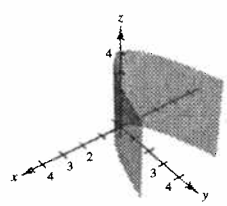
10. $x^2 + z^2 = 25$

The y-coordinate is missing so we have a cylindrical surface with rulings parallel to the y-axis. The generating curve is a circle.



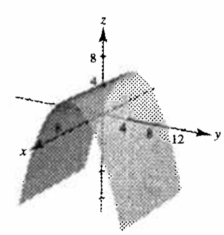
11. $y = x^2$

The z-coordinate is missing so we have a cylindrical surface with rulings parallel to the z-axis. The generating curve is a parabola.



12. $z = 4 - y^2$

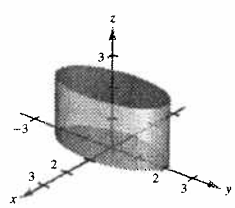
The x-coordinate is missing so we have a cylindrical surface with rulings parallel to the x-axis. The generating curve is a parabola.



13. $4x^2 + y^2 = 4$

$$\frac{x^2}{1} + \frac{y^2}{4} = 1$$

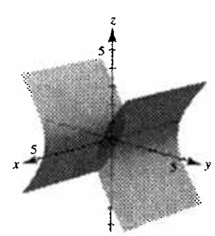
The z-coordinate is missing so we have a cylindrical surface with rulings parallel to the z-axis. The generating curve is an ellipse.



14. $y^2 - z^2 = 4$

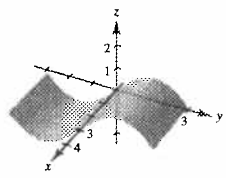
$$\frac{y^2}{4} - \frac{z^2}{4} = 1$$

The x-coordinate is missing so we have a cylindrical surface with rulings parallel to the x-axis. The generating curve is a hyperbola.



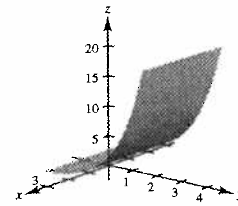
15. $z = \sin y$

The x-coordinate is missing so we have a cylindrical surface with rulings parallel to the x-axis. The generating curve is the sine curve.



16. $z = e^y$

The x-coordinate is missing so we have a cylindrical surface with rulings parallel to the x-axis. The generating curve is the exponential curve.



17. $z = x^2 + y^2$

(a) You are viewing the paraboloid from the x-axis: $(20, 0, 0)$

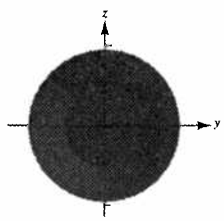
(b) You are viewing the paraboloid from above, but not on the z-axis: $(10, 10, 20)$

(c) You are viewing the paraboloid from the z-axis: $(0, 0, 20)$

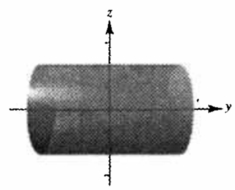
(d) You are viewing the paraboloid from the y-axis: $(0, 20, 0)$

18. $y^2 + z^2 = 4$

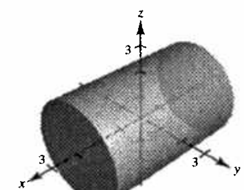
(a) From $(10, 0, 0)$:



(b) From $(0, 10, 0)$:



(c) From $(10, 10, 10)$:



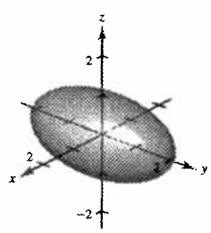
19. $\dfrac{x^2}{1} + \dfrac{y^2}{4} + \dfrac{z^2}{1} = 1$

Ellipsoid

xy-trace: $\dfrac{x^2}{1} + \dfrac{y^2}{4} = 1$ ellipse

xz-trace: $x^2 + z^2 = 1$ circle

yz-trace: $\dfrac{y^2}{4} + \dfrac{z^2}{1} = 1$ ellipse



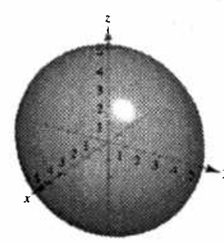
20. $\dfrac{x^2}{16} + \dfrac{y^2}{25} + \dfrac{z^2}{25} = 1$

Ellipsoid

xy-trace: $\dfrac{x^2}{16} + \dfrac{y^2}{25} = 1$ ellipse

xz-trace: $\dfrac{x^2}{16} + \dfrac{z^2}{25} = 1$ ellipse

yz-trace: $y^2 + z^2 = 25$ circle



21. $16x^2 - y^2 + 16z^2 = 4$

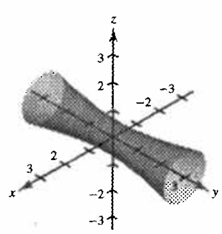
$4x^2 - \dfrac{y^2}{4} + 4z^2 = 1$

Hyperboloid on one sheet

xy-trace: $4x^2 - \dfrac{y^2}{4} = 1$ hyperbola

xz-trace: $4(x^2 + z^2) = 1$ circle

yz-trace: $\dfrac{-y^2}{4} + 4z^2 = 1$ hyperbola



22. $z^2 - x^2 - \dfrac{y^2}{4} = 1$

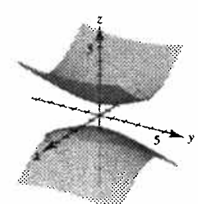
Hyperboloid of two sheets

xy-trace: none

xz-trace: $z^2 - x^2 = 1$ hyperbola

yz-trace: $z^2 - \dfrac{y^2}{4} = 1$ hyperbola

$z = \pm\sqrt{10}$: $\dfrac{x^2}{9} + \dfrac{y^2}{36} = 1$ ellipse



23. $x^2 - y + z^2 = 0$

Elliptic paraboloid

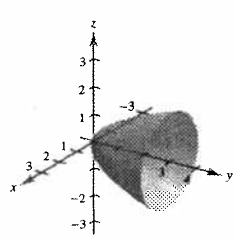
xy-trace: $y = x^2$

xz-trace: $x^2 + z^2 = 0$,

point $(0, 0, 0)$

yz-trace: $y = z^2$

$y = 1$: $x^2 + z^2 = 1$



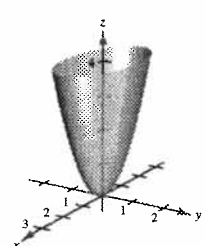
24. $z = x^2 + 4y^2$

Elliptic paraboloid

xy-trace: point $(0, 0, 0)$

xz-trace: $z = x^2$ parabola

yz-trace: $z = 4y^2$ parabola



25. $x^2 - y^2 + z = 0$

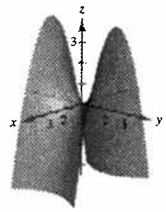
Hyperbolic paraboloid

xy-trace: $y = \pm x$

xz-trace: $z = -x^2$

yz-trace: $z = y^2$

$y = \pm 1$: $z = 1 - x^2$



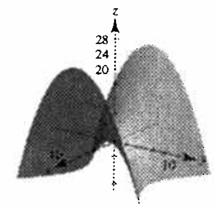
26. $3z = -y^2 + x^2$

Hyperbolic paraboloid

xy-trace: $y = \pm x$

xz-trace: $z = \frac{1}{3}x^2$

yz-trace: $z = -\frac{1}{3}y^2$



27. $z^2 = x^2 + \dfrac{y^2}{4}$

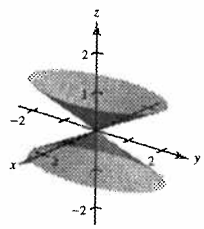
Elliptic Cone

xy-trace: point $(0, 0, 0)$

xz-trace: $z = \pm x$

yz-trace: $z = \dfrac{\pm 1}{2}y$

$z = \pm 1$: $x^2 + \dfrac{y^2}{4} = 1$



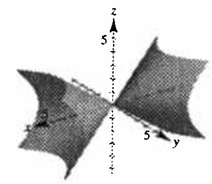
28. $x^2 = 2y^2 + 2z^2$

Elliptic Cone

xy-trace: $x = \pm\sqrt{2}y$

xz-trace: $x = \pm\sqrt{2}z$

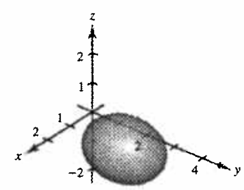
yz-trace: point: $(0, 0, 0)$



29.

$$16x^2 + 9y^2 + 16z^2 - 32x - 36y + 36 = 0$$
$$16(x^2 - 2x + 1) + 9(y^2 - 4y + 4) + 16z^2 = -36 + 16 + 36$$
$$16(x - 1)^2 + 9(y - 2)^2 + 16z^2 = 16$$
$$\frac{(x - 1)^2}{1} + \frac{(y - 2)^2}{16/9} + \frac{z^2}{1} = 1$$

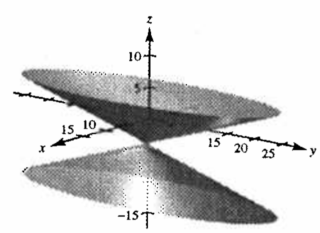
Ellipsoid with center $(1, 2, 0)$.



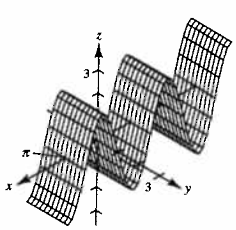
30.

$$9x^2 + y^2 - 9z^2 - 54x - 4y - 54z + 4 = 0$$
$$9(x^2 - 6x + 9) + (y^2 - 4y + 4) - 9(z^2 + 6z + 9) = 81 - 81$$
$$9(x - 3)^2 + (y - 2)^2 - 9(z + 3)^2 = 0$$

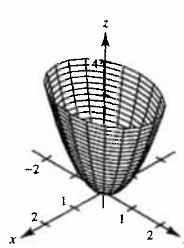
Elliptic cone with center $(3, 2, -3)$.



31. $z = 2 \sin x$

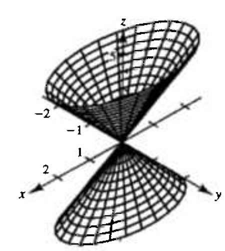


32. $z = x^2 + 0.5y^2$



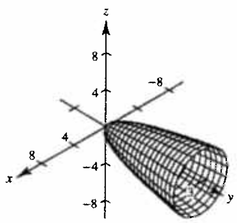
33. $z^2 = x^2 + 4y^2$

$z = \pm\sqrt{x^2 + 4y^2}$



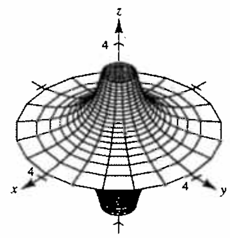
34. $z^2 = 4y - x^2$

$z = \pm\sqrt{4y - x^2}$



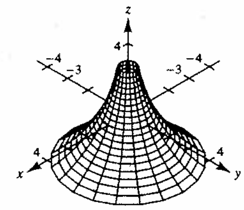
35. $x^2 + y^2 = \left(\frac{2}{z}\right)^2$

$y = \pm\sqrt{\frac{4}{z^2} - x^2}$

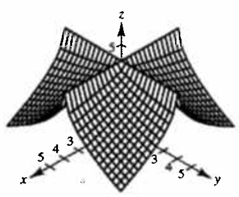


36. $x^2 + y^2 = e^{-z}$

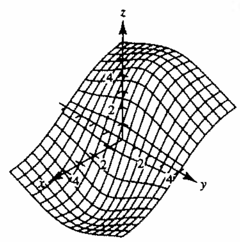
$-\ln(x^2 + y^2) = z$



37. $z = 4 - \sqrt{|xy|}$

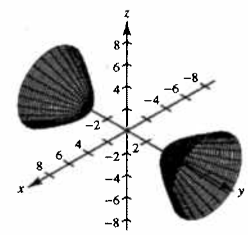


38. $z = \dfrac{-x}{8 + x^2 + y^2}$



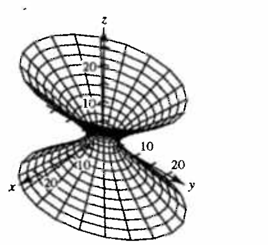
39. $4x^2 - y^2 + 4z^2 = -16$

$z = \pm\sqrt{\frac{y^2}{4} - x^2 - 4}$



40. $9x^2 + 4y^2 - 8z^2 = 72$

$z = \pm\sqrt{\frac{9}{8}x^2 + \frac{1}{2}y^2 - 9}$

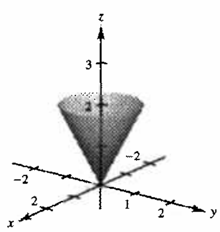


41. $z = 2\sqrt{x^2 + y^2}$

$z = 2$

$2\sqrt{x^2 + y^2} = 2$

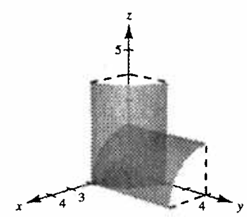
$x^2 + y^2 = 1$



42. $z = \sqrt{4 - x^2}$

$y = \sqrt{4 - x^2}$

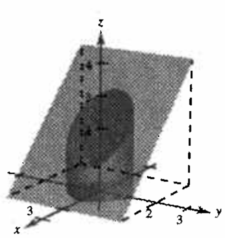
$x = 0, y = 0, z = 0$



43. $x^2 + y^2 = 1$

$x + z = 2$

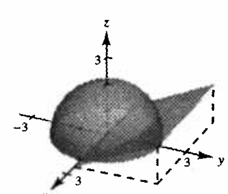
$z = 0$



44. $z = \sqrt{4 - x^2 - y^2}$

$y = 2z$

$z = 0$



45. $x^2 + z^2 = [r(y)]^2$ and $z = r(y) = \pm 2\sqrt{y}$; therefore,

$x^2 + z^2 = 4y.$

46. $x^2 + z^2 = [r(y)]^2$ and $z = r(y) = 3y$; therefore,

$x^2 + z^2 = 9y^2.$

47. $x^2 + y^2 = [r(z)]^2$ and $y = r(z) = \frac{z}{2}$; therefore,

$x^2 + y^2 = \frac{z^2}{4}, 4x^2 + 4y^2 = z^2.$

48. $y^2 + z^2 = [r(x)]^2$ and $z = r(x) = \frac{1}{2}\sqrt{4 - x^2}$; therefore,

$y^2 + z^2 = \frac{1}{4}(4 - x^2), x^2 + 4y^2 + 4z^2 = 4.$

49. $y^2 + z^2 = [r(x)]^2$ and $y = r(x) = \dfrac{2}{x}$; therefore,

$$y^2 + z^2 = \left(\frac{2}{x}\right)^2, y^2 + z^2 = \frac{4}{x^2}.$$

50. $x^2 + y^2 = [r(z)]^2$ and $y = r(z) = e^z$; therefore,

$$x^2 + y^2 = e^{2z}.$$

51. $x^2 + y^2 - 2z = 0$

$x^2 + y^2 = \left(\sqrt{2z}\right)^2$

Equation of generating curve: $y = \sqrt{2z}$ or $x = \sqrt{2z}$

52. $x^2 + z^2 = \cos^2 y$

Equation of generating curve: $x = \cos y$ or $z = \cos y$

53. Let C be a curve in a plane and let L be a line not in a parallel plane. The set of all lines parallel to L and intersecting C is called a cylinder.

54. The trace of a surface is the intersection of the surface with a plane. You find a trace by setting one variable equal to a constant, such as $x = 0$ or $z = 2$.

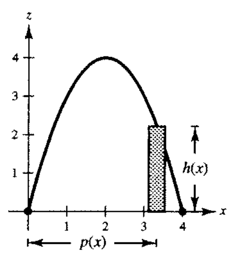
55. See pages 812 and 813.

56. In the xz-plane, $z = x^2$ is a parabola.

In three-space, $z = x^2$ is a cylinder.

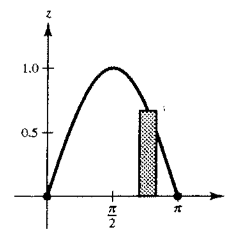
57. $V = 2\pi \displaystyle\int_0^4 x(4x - x^2)\,dx$

$$= 2\pi\left[\frac{4x^3}{3} - \frac{x^4}{4}\right]_0^4 = \frac{128\pi}{3}$$



58. $V = 2\pi \displaystyle\int_0^{\pi} y \sin y\,dy$

$$= 2\pi\Big[\sin y - y\cos y\Big]_0^{\pi} = 2\pi^2$$



59. $z = \dfrac{x^2}{2} + \dfrac{y^2}{4}$

(a) When $z = 2$ we have $2 = \dfrac{x^2}{2} + \dfrac{y^2}{4}$, or $1 = \dfrac{x^2}{4} + \dfrac{y^2}{8}$

Major axis: $2\sqrt{8} = 4\sqrt{2}$

Minor axis: $2\sqrt{4} = 4$

$c^2 = a^2 - b^2, c^2 = 4, c = 2$

Foci: $(0, \pm 2, 2)$

(b) When $z = 8$ we have $8 = \dfrac{x^2}{2} + \dfrac{y^2}{4}$, or $1 = \dfrac{x^2}{16} + \dfrac{y^2}{32}$.

Major axis: $2\sqrt{32} = 8\sqrt{2}$

Minor axis: $2\sqrt{16} = 8$

$c^2 = 32 - 16 = 16, c = 4$

Foci: $(0, \pm 4, 8)$

60. $z = \dfrac{x^2}{2} + \dfrac{y^2}{4}$

(a) When $y = 4$ we have $z = \dfrac{x^2}{2} + 4, 4\left(\dfrac{1}{2}\right)(z - 4) = x^2$.

Focus: $\left(0, 4, \dfrac{9}{2}\right)$

(b) When $x = 2$ we have

$z = 2 + \dfrac{y^2}{4}, 4(z - 2) = y^2$.

Focus: $(2, 0, 3)$

61. If (x, y, z) is on the surface, then

$$(y + 2)^2 = x^2 + (y - 2)^2 + z^2$$

$$y^2 + 4y + 4 = x^2 + y^2 - 4y + 4 + z^2$$

$$x^2 + z^2 = 8y$$

Elliptic paraboloid

Traces parallel to xz-plane are circles.

62. If (x, y, z) is on the surface, then

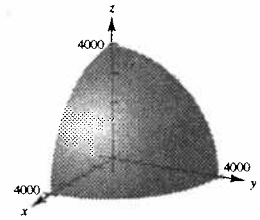
$$z^2 = x^2 + y^2 + (z - 4)^2$$

$$z^2 = x^2 + y^2 + z^2 - 8z + 16$$

$$8z = x^2 + y^2 + 16 \Longrightarrow z = \frac{x^2}{8} + \frac{y^2}{8} + 2$$

Elliptic paraboloid shifted up 2 units. Traces parallel to xy-plane are circles.

63. $\dfrac{x^2}{3963^2} + \dfrac{y^2}{3963^2} + \dfrac{z^2}{3950^2} = 1$

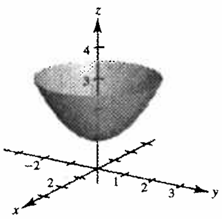


64. (a)

$$x^2 + y^2 = [r(z)]^2$$

$$= \left[\sqrt{2(z-1)}\right]^2$$

$$x^2 + y^2 - 2z + 2 = 0$$

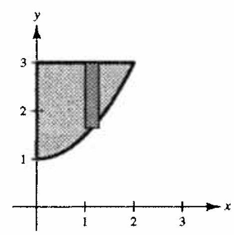


(b) $V = 2\pi\int_0^2 x\left[3 - \left(\frac{1}{2}x^2 + 1\right)\right] dx$

$$= 2\pi\int_0^2 \left(2x - \frac{1}{2}x^3\right) dx$$

$$= 2\pi\left[x^2 - \frac{x^4}{8}\right]_0^2$$

$$= 4\pi \approx 12.6 \text{ cm}^3$$

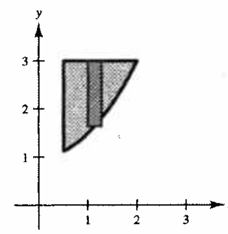


(c) $V = 2\pi\int_{1/2}^2 x\left[3 - \left(\frac{1}{2}x^2 + 1\right)\right] dx$

$$= 2\pi\int_{1/2}^2 \left(2x - \frac{1}{2}x^3\right) dx$$

$$= 2\pi\left[x^2 - \frac{x^4}{8}\right]_{1/2}^2$$

$$= 4\pi - \frac{31\pi}{64} = \frac{225\pi}{64} \approx 11.04 \text{ cm}^3$$



65. $z = \dfrac{y^2}{b^2} - \dfrac{x^2}{a^2}$, $z = bx + ay$

$$bx + ay = \frac{y^2}{b^2} - \frac{x^2}{a^2}$$

$$\frac{1}{a^2}\left(x^2 + a^2bx + \frac{a^4b^2}{4}\right) = \frac{1}{b^2}\left(y^2 - ab^2y + \frac{a^2b^4}{4}\right)$$

$$\frac{\left(x + \frac{a^2b}{2}\right)^2}{a^2} = \frac{\left(y - \frac{ab^2}{2}\right)^2}{b^2}$$

$$y = \pm\frac{b}{a}\left(x + \frac{a^2b}{2}\right) + \frac{ab^2}{2}$$

Letting $x = at$, you obtain the two intersecting lines $x = at$, $y = -bt$, $z = 0$ and $x = at$, $y = bt + ab^2$ $z = 2abt + a^2b^2$.

66. Equating twice the first equation with the second equation,

$$2x^2 + 6y^2 - 4z^2 + 4y - 8 = 2x^2 + 6y^2 - 4z^2 - 3x - 2$$

$$4y - 8 = -3x - 2$$

$$3x + 4y = 6, \text{ a plane}$$

67. True. A sphere is a special case of an ellipsoid (centered at origin, for example)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

having $a = b = c$.

68. False. For example, the surface $x^2 + z^2 = e^{-2y}$ can be formed by revolving the graph of $x = e^{-y}$ about the y-axis, as the graph of $z = e^{-y}$ about the y-axis.

69. The Klein bottle *does not* have both an "inside" and an "outside." It is formed by inserting the small open end through the side of the bottle and making it contiguous with the top of the bottle.

Section 11.7 Cylindrical and Spherical Coordinates

1. $(5, 0, 2)$, cylindrical

$x = 5\cos 0 = 5$

$y = 5\sin 0 = 0$

$z = 2$

$(5, 0, 2)$, rectangular

2. $\left(4, \frac{\pi}{2}, -2\right)$, cylindrical

$x = 4\cos\frac{\pi}{2} = 0$

$y = 4\sin\frac{\pi}{2} = 4$

$z = -2$

$(0, 4, -2)$, rectangular

3. $\left(2, \frac{\pi}{3}, 2\right)$, cylindrical

$x = 2\cos\frac{\pi}{3} = 1$

$y = 2\sin\frac{\pi}{3} = \sqrt{3}$

$z = 2$

$\left(1, \sqrt{3}, 2\right)$, rectangular

4. $\left(6, -\frac{\pi}{4}, 2\right)$, cylindrical

$x = 6\cos\left(-\frac{\pi}{4}\right) = 3\sqrt{2}$

$y = 6\sin\left(-\frac{\pi}{4}\right) = -3\sqrt{2}$

$z = 2$

$\left(3\sqrt{2}, -3\sqrt{2}, 2\right)$

5. $\left(4, \frac{7\pi}{6}, 3\right)$, cylindrical

$x = 4\cos\frac{7\pi}{6} = -2\sqrt{3}$

$y = 4\sin\frac{7\pi}{6} = -2$

$z = 3$

$\left(-2\sqrt{3}, -2, 3\right)$, rectangular

6. $\left(1, \frac{3\pi}{2}, 1\right)$, cylindrical

$x = \cos\frac{3\pi}{2} = 0$

$y = \sin\frac{3\pi}{2} = -1$

$z = 1$

$(0, -1, 1)$, rectangular

7. $(0, 5, 1)$, rectangular

$r = \sqrt{(0)^2 + (5)^2} = 5$

$\theta = \arctan\frac{5}{0} = \frac{\pi}{2}$

$z = 1$

$\left(5, \frac{\pi}{2}, 1\right)$, cylindrical

8. $\left(2\sqrt{2}, -2\sqrt{2}, 4\right)$, rectangular

$r = \sqrt{\left(2\sqrt{2}\right)^2 + \left(-2\sqrt{2}\right)^2} = 4$

$\theta = \arctan(-1) = -\frac{\pi}{4}$

$z = 4$

$\left(4, -\frac{\pi}{4}, 4\right)$, cylindrical

9. $\left(1, \sqrt{3}, 4\right)$, rectangular

$r = \sqrt{1^2 + \left(\sqrt{3}\right)^2} = 2$

$\theta = \arctan\sqrt{3} = \frac{\pi}{3}$

$z = 4$

$\left(2, \frac{\pi}{3}, 4\right)$, cylindrical

10. $\left(2\sqrt{3}, -2, 6\right)$, rectangular

$r = \sqrt{12 + 4} = 4$

$\theta = \arctan\left(-\frac{1}{\sqrt{3}}\right) = \frac{5\pi}{6}$

$z = 6$

$\left(4, -\frac{\pi}{6}, 6\right)$, cylindrical

11. $(2, -2, -4)$, rectangular

$r = \sqrt{2^2 + (-2)^2} = 2\sqrt{2}$

$\theta = \arctan(-1) = -\frac{\pi}{4}$

$z = -4$

$\left(2\sqrt{2}, \frac{-\pi}{4}, -4\right)$, cylindrical

12. $(-3, 2, -1)$, rectangular

$r = \sqrt{(-3)^2 + 2^2} = \sqrt{13}$

$\theta = \arctan\left(\frac{-2}{3}\right) = -\arctan\frac{2}{3}$

$z = -1$

$\left(\sqrt{13}, -\arctan\frac{2}{3}, -1\right)$, cylindrical

13. $z = 5$ is the equation in cylindrical coordinates.

14. $x = 4$, rectangular coordinates

$r\cos\theta = 4$

$r = 4\sec\theta$, cylindrical coordinates

15. $x^2 + y^2 + z^2 = 10$, rectangular equation

$r^2 + z^2 = 10$, cylindrical equation

16. $z = x^2 + y^2 - 2$, rectangular equation

$z = r^2 - 2$, cylindrical equation

17. $y = x^2$, rectangular equation

$r\sin\theta = (r\cos\theta)^2$

$\sin\theta = r\cos^2\theta$

$r = \sec\theta \cdot \tan\theta$, cylindrical equation

18. $x^2 + y^2 = 8x$, rectangular equation

$r^2 = 8r\cos\theta$

$r = 8\cos\theta$, cylindrical equation

19. $y^2 = 10 - z^2$, rectangular coordinates

$(r \sin \theta)^2 = 10 - z^2$

$r^2 \sin^2 \theta + z^2 = 10$, cylindrical coordinates

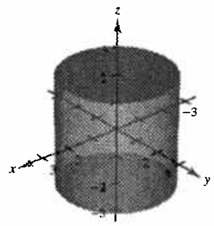
20. $x^2 + y^2 + z^2 - 3z = 0$, rectangular coordinates

$r^2 + z^2 - 3z = 0$ cylindrical coordinates

21. $r = 2$

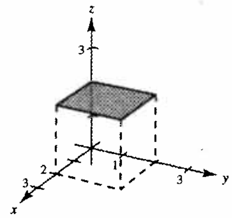
$\sqrt{x^2 + y^2} = 2$

$x^2 + y^2 = 4$



22. $z = 2$

Same



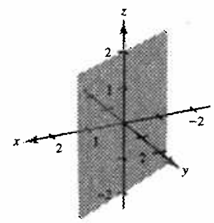
23. $\theta = \dfrac{\pi}{6}$

$\tan \dfrac{\pi}{6} = \dfrac{y}{x}$

$\dfrac{1}{\sqrt{3}} = \dfrac{y}{x}$

$x = \sqrt{3}y$

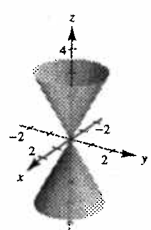
$x - \sqrt{3}y = 0$



24. $r = \dfrac{z}{2}$

$\sqrt{x^2 + y^2} = \dfrac{z}{2}$

$x^2 + y^2 - \dfrac{z^2}{4} = 0$



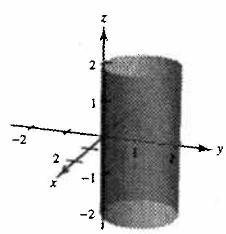
25. $r = 2 \sin \theta$

$r^2 = 2r \sin \theta$

$x^2 + y^2 = 2y$

$x^2 + y^2 - 2y = 0$

$x^2 + (y - 1)^2 = 1$



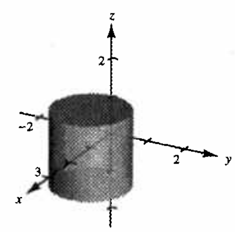
26. $r = 2 \cos \theta$

$r^2 = 2r \cos \theta$

$x^2 + y^2 = 2x$

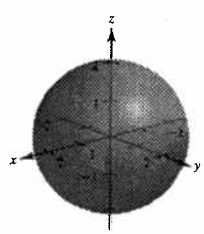
$x^2 + y^2 - 2x = 0$

$(x - 1)^2 + y^2 = 1$



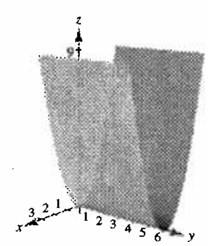
27. $r^2 + z^2 = 4$

$x^2 + y^2 + z^2 = 4$



28. $z = r^2 \cos^2 \theta$

$z = x^2$



29. $(4, 0, 0)$, rectangular

$\rho = \sqrt{4^2 + 0^2 + 0^2} = 4$

$\theta = \arctan 0 = 0$

$\phi = \arccos 0 = \dfrac{\pi}{2}$

$\left(4, 0, \dfrac{\pi}{2}\right)$, spherical

30. $(1, 1, 1)$, rectangular

$\rho = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

$\theta = \arctan 1 = \dfrac{\pi}{4}$

$\phi = \arccos \dfrac{1}{\sqrt{3}}$

$\left(\sqrt{3}, \dfrac{\pi}{4}, \arccos \dfrac{1}{\sqrt{3}}\right)$, spherical

31. $(-2, 2\sqrt{3}, 4)$, rectangular

$$\rho = \sqrt{(-2)^2 + (2\sqrt{3})^2 + 4^2} = 4\sqrt{2}$$

$$\theta = \arctan(-\sqrt{3}) = \frac{2\pi}{3}$$

$$\phi = \arccos\frac{1}{\sqrt{2}} = \frac{\pi}{4}$$

$\left(4\sqrt{2}, \frac{2\pi}{3}, \frac{\pi}{4}\right)$, spherical

32. $(2, 2, 4\sqrt{2})$, rectangular

$$\rho = \sqrt{2^2 + 2^2 + (4\sqrt{2})^2} = 2\sqrt{10}$$

$$\theta = \arctan 1 = \frac{\pi}{4}$$

$$\phi = \arccos\frac{2}{\sqrt{5}}$$

$\left(2\sqrt{10}, \frac{\pi}{4}, \arccos\frac{2}{\sqrt{5}}\right)$, spherical

33. $(\sqrt{3}, 1, 2\sqrt{3})$, rectangular

$$\rho = \sqrt{3 + 1 + 12} = 4$$

$$\theta = \arctan\frac{1}{\sqrt{3}} = \frac{\pi}{6}$$

$$\phi = \arccos\frac{\sqrt{3}}{2} = \frac{\pi}{6}$$

$\left(4, \frac{\pi}{6}, \frac{\pi}{6}\right)$, spherical

34. $(-4, 0, 0)$, rectangular

$$\rho = \sqrt{(-4)^2 + 0^2 + 0^2} = 4$$

$$\theta = \pi$$

$$\phi = \arccos 0 = \frac{\pi}{2}$$

$\left(4, \pi, \frac{\pi}{2}\right)$, spherical

35. $\left(4, \frac{\pi}{6}, \frac{\pi}{4}\right)$, spherical

$$x = 4\sin\frac{\pi}{4}\cos\frac{\pi}{6} = \sqrt{6}$$

$$y = 4\sin\frac{\pi}{4}\sin\frac{\pi}{6} = \sqrt{2}$$

$$z = 4\cos\frac{\pi}{4} = 2\sqrt{2}$$

$(\sqrt{6}, \sqrt{2}, 2\sqrt{2})$, rectangular

36. $\left(12, \frac{3\pi}{4}, \frac{\pi}{9}\right)$, spherical

$$x = 12\sin\frac{\pi}{9}\cos\frac{3\pi}{4} \approx -2.902$$

$$y = 12\sin\frac{\pi}{9}\sin\frac{3\pi}{4} \approx 2.902$$

$$z = 12\cos\frac{\pi}{9} \approx 11.276$$

$(-2.902, 2.902, 11.276)$, rectangular

37. $\left(12, \frac{-\pi}{4}, 0\right)$, spherical

$$x = 12\sin 0\cos\left(\frac{-\pi}{4}\right) = 0$$

$$y = 12\sin 0\sin\left(\frac{-\pi}{4}\right) = 0$$

$$z = 12\cos 0 = 12$$

$(0, 0, 12)$, rectangular

38. $\left(9, \frac{\pi}{4}, \pi\right)$, spherical

$$x = 9\sin\pi\cos\frac{\pi}{4} = 0$$

$$y = 9\sin\pi\sin\frac{\pi}{4} = 0$$

$$z = 9\cos\pi = -9$$

$(0, 0, -9)$, rectangular

39. $\left(5, \frac{\pi}{4}, \frac{3\pi}{4}\right)$, spherical

$$x = 5\sin\frac{3\pi}{4}\cos\frac{\pi}{4} = \frac{5}{2}$$

$$y = 5\sin\frac{3\pi}{4}\sin\frac{\pi}{4} = \frac{5}{2}$$

$$z = 5\cos\frac{3\pi}{4} = -\frac{5\sqrt{2}}{2}$$

$\left(\frac{5}{2}, \frac{5}{2}, -\frac{5\sqrt{2}}{2}\right)$, rectangular

40. $\left(6, \pi, \frac{\pi}{2}\right)$, spherical

$$x = 6\sin\frac{\pi}{2}\cos\pi = -6$$

$$y = 6\sin\frac{\pi}{2}\sin\pi = 0$$

$$z = 6\cos\frac{\pi}{2} = 0$$

$(-6, 0, 0)$, rectangular

41. $y = 3$, rectangular equation

$\rho\sin\phi\sin\theta = 3$

$\rho = 3\csc\phi\csc\theta$, spherical equation

42. $z = 2$, rectangular equation

$\rho\cos\phi = 2$

$\rho = 2\sec\phi$, pherical equation

43. $x^2 + y^2 + z^2 = 36$, rectangular equation

$$\rho^2 = 36$$

$$\rho = 6, \quad \text{spherical equation}$$

44. $x^2 + y^2 - 3z^2 = 0$, rectangular equation

$$x^2 + y^2 + z^2 = 4z^2$$

$$\rho^2 = 4\rho^2 \cos^2 \phi$$

$$1 = 4\cos^2 \phi$$

$$\cos \phi = \frac{1}{2}$$

$$\phi = \frac{\pi}{3}, \quad \text{(cone) spherical equation}$$

45. $x^2 + y^2 = 9$, rectangular equation

$$\rho^2 \sin^2 \phi \cos^2 \theta + \rho^2 \sin^2 \phi \sin^2 \theta = 9$$

$$\rho^2 \sin^2 \phi = 9$$

$$\rho \sin \phi = 3$$

$$\rho = 3 \csc \phi, \quad \text{spherical equation}$$

46. $x = 10$, rectangular equation

$$\rho \sin \phi \cos \theta = 10$$

$$\rho = 10 \csc \phi \sec \theta, \quad \text{spherical equation}$$

47. $x^2 + y^2 = 2z^2$ rectangular coordinates

$$\rho^2 \sin^2 \phi \cos^2 \theta + \rho^2 \sin^2 \phi \sin^2 \theta = 2\rho^2 \cos^2 \phi$$

$$\rho^2 \sin^2 \phi[\cos^2 \theta + \sin^2 \theta] = 2\rho^2 \cos^2 \phi$$

$$\rho^2 \sin^2 \phi = 2\rho^2 \cos^2 \phi$$

$$\frac{\sin^2 \phi}{\cos^2 \phi} = 2$$

$$\tan^2 \phi = 2$$

$$\tan \phi = \pm\sqrt{2} \quad \text{spherical equation}$$

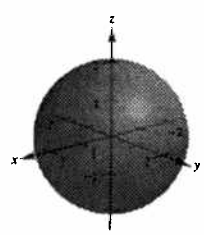
48. $x^2 + y^2 + z^2 - 9z = 0$ rectangular equation

$$\rho^2 - 9\rho \cos \phi = 0$$

$$\rho = 9 \cos \phi \quad \text{spherical equation}$$

49. $\rho = 2$

$$x^2 + y^2 + z^2 = 4$$

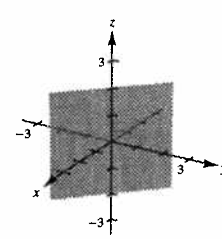


50. $\theta = \dfrac{3\pi}{4}$

$$\tan \theta = \frac{y}{x}$$

$$-1 = \frac{y}{x}$$

$$x + y = 0$$



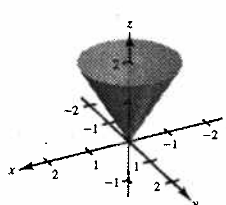
51. $\phi = \dfrac{\pi}{6}$

$$\cos \phi = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$\frac{\sqrt{3}}{2} = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$\frac{3}{4} = \frac{z^2}{x^2 + y^2 + z^2}$$

$$3x^2 + 3y^2 - z^2 = 0, z \geq 0$$



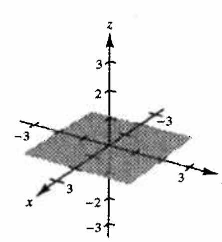
52. $\phi = \dfrac{\pi}{2}$

$$\cos \phi = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$0 = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$z = 0$$

xy-plane

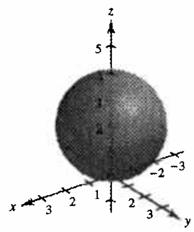


53. $\rho = 4\cos\phi$

$$\sqrt{x^2 + y^2 + z^2} = \frac{4z}{\sqrt{x^2 + y^2 + z^2}}$$

$$x^2 + y^2 + z^2 - 4z = 0$$

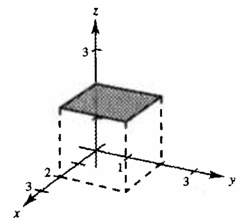
$$x^2 + y^2 + (z - 2)^2 = 4, z \geq 0$$



54. $\rho = 2\sec\phi$

$$\rho\cos\phi = 2$$

$$z = 2$$

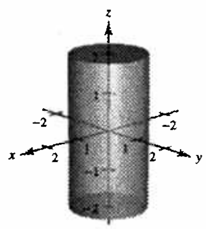


55. $\rho = \csc\phi$

$$\rho\sin\phi = 1$$

$$\sqrt{x^2 + y^2} = 1$$

$$x^2 + y^2 = 1$$

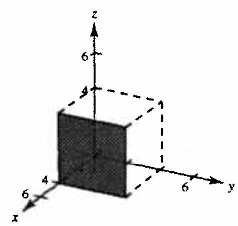


56. $\rho = 4\csc\phi\sec\phi$

$$= \frac{4}{\sin\phi\cos\theta}$$

$$\rho\sin\phi\cos\theta = 4$$

$$x = 4$$



57. $\left(4, \frac{\pi}{4}, 0\right)$, cylindrical

$$\rho = \sqrt{4^2 + 0^2} = 4$$

$$\theta = \frac{\pi}{4}$$

$$\phi = \arccos 0 = \frac{\pi}{2}$$

$\left(4, \frac{\pi}{4}, \frac{\pi}{2}\right)$, spherical

58. $\left(3, -\frac{\pi}{4}, 0\right)$, cylindrical

$$\rho = \sqrt{3^2 + 0^2} = 3$$

$$\theta = -\frac{\pi}{4}$$

$$\phi = \arccos\left(\frac{0}{9}\right) = \frac{\pi}{2}$$

$\left(3, -\frac{\pi}{4}, \frac{\pi}{2}\right)$, spherical

59. $\left(4, \frac{\pi}{2}, 4\right)$, cylindrical

$$\rho = \sqrt{4^2 + 4^2} = 4\sqrt{2}$$

$$\theta = \frac{\pi}{2}$$

$$\phi = \arccos\left(\frac{4}{4\sqrt{2}}\right) = \frac{\pi}{4}$$

$\left(4\sqrt{2}, \frac{\pi}{2}, \frac{\pi}{4}\right)$, spherical

60. $\left(2, \frac{2\pi}{3}, -2\right)$, cylindrical

$$\rho = \sqrt{2^2 + (-2)^2} = 2\sqrt{2}$$

$$\theta = \frac{2\pi}{3}$$

$$\phi = \arccos\left(\frac{-1}{\sqrt{2}}\right) = \frac{3\pi}{4}$$

$\left(2\sqrt{2}, \frac{2\pi}{3}, \frac{3\pi}{4}\right)$, spherical

61. $\left(4, \frac{-\pi}{6}, 6\right)$, cylindrical

$$\rho = \sqrt{4^2 + 6^2} = 2\sqrt{13}$$

$$\theta = \frac{-\pi}{6}$$

$$\phi = \arccos\frac{3}{\sqrt{13}}$$

$\left(2\sqrt{13}, \frac{-\pi}{6}, \arccos\frac{3}{\sqrt{13}}\right)$, spherical

62. $\left(-4, \frac{\pi}{3}, 4\right)$, cylindrical

$$\rho = \sqrt{(-4)^2 + 4^2} = 4\sqrt{2}$$

$$\theta = \frac{\pi}{3}$$

$$\phi = \arccos\frac{1}{\sqrt{2}} = \frac{\pi}{4}$$

$\left(4\sqrt{2}, \frac{\pi}{3}, \frac{\pi}{4}\right)$, spherical

63. $(12, \pi, 5)$, cylindrical

$$\rho = \sqrt{12^2 + 5^2} = 13$$

$$\theta = \pi$$

$$\phi = \arccos\frac{5}{13}$$

$\left(13, \pi, \arccos\frac{5}{13}\right)$, spherical

64. $\left(4, \frac{\pi}{2}, 3\right)$, cylindrical

$$\rho = \sqrt{4^2 + 3^2} = 5$$

$$\theta = \frac{\pi}{2}$$

$$\phi = \arccos\frac{3}{5}$$

$\left(5, \frac{\pi}{2}, \arccos\frac{3}{5}\right)$, spherical

65. $\left(10, \frac{\pi}{6}, \frac{\pi}{2}\right)$, spherical

$$r = 10\sin\frac{\pi}{2} = 10$$

$$\theta = \frac{\pi}{6}$$

$$z = 10\cos\frac{\pi}{2} = 0$$

$\left(10, \frac{\pi}{6}, 0\right)$, cylindrical

66. $\left(4, \frac{\pi}{18}, \frac{\pi}{2}\right)$, spherical

$$r = 4\sin\frac{\pi}{2} = 4$$

$$\theta = \frac{\pi}{18}$$

$$z = 4\cos\frac{\pi}{2} = 0$$

$\left(4, \frac{\pi}{18}, 0\right)$, cylindrical

67. $\left(36, \pi, \frac{\pi}{2}\right)$, spherical

$r = \rho \sin \phi = 36 \sin \frac{\pi}{2} = 36$

$\theta = \pi$

$z = \rho \cos \phi = 36 \cos \frac{\pi}{2} = 0$

$(36, \pi, 0)$, cylindrical

68. $\left(18, \frac{\pi}{3}, \frac{\pi}{3}\right)$, spherical

$r = \rho \sin \phi = 18 \sin \frac{\pi}{3} = 9$

$\theta = \frac{\pi}{3}$

$z = \rho \cos \phi = 18 \cos \frac{\pi}{3} = 9\sqrt{3}$

$\left(9, \frac{\pi}{3}, 9\sqrt{3}\right)$, cylindrical

69. $\left(6, -\frac{\pi}{6}, \frac{\pi}{3}\right)$, spherical

$r = 6 \sin \frac{\pi}{3} = 3\sqrt{3}$

$\theta = -\frac{\pi}{6}$

$z = 6 \cos \frac{\pi}{3} = 3$

$\left(3\sqrt{3}, -\frac{\pi}{6}, 3\right)$, cylindrical

70. $\left(5, -\frac{5\pi}{6}, \pi\right)$, spherical

$r = 5 \sin \pi = 0$

$\theta = -\frac{5\pi}{6}$

$z = 5 \cos \pi = -5$

$\left(0, -\frac{5\pi}{6}, -5\right)$, cylindrical

71. $\left(8, \frac{7\pi}{6}, \frac{\pi}{6}\right)$, spherical

$r = 8 \sin \frac{\pi}{6} = 4$

$\theta = \frac{7\pi}{6}$

$z = 8 \cos \frac{\pi}{6} = \frac{8\sqrt{3}}{2}$

$\left(4, \frac{7\pi}{6}, 4\sqrt{3}\right)$, cylindrical

72. $\left(7, \frac{\pi}{4}, \frac{3\pi}{4}\right)$, spherical

$r = 7 \sin \frac{3\pi}{4} = \frac{7\sqrt{2}}{2}$

$\theta = \frac{\pi}{4}$

$z = 7 \cos \frac{3\pi}{4} = -\frac{7\sqrt{2}}{2}$

$\left(\frac{7\sqrt{2}}{2}, \frac{\pi}{4}, -\frac{7\sqrt{2}}{2}\right)$, cylindrical

	Rectangular	Cylindrical	Spherical
73.	$(4, 6, 3)$	$(7.211, 0.983, 3)$	$(7.810, 0.983, 1.177)$
74.	$(6, -2, -3)$	$(6.325, -0.322, -3)$	$(7.000, -0.322, 2.014)$
75.	$(4.698, 1.710, 8)$	$\left(5, \frac{\pi}{9}, 8\right)$	$(9.434, 0.349, 0.559)$
76.	$(7.317, -6.816, 6)$	$(10, -0.75, 6)$	$(11.662, -0.750, 1.030)$
77.	$(-7.071, 12.247, 14.142)$	$(14.142, 2.094, 14.142)$	$\left(20, \frac{2\pi}{3}, \frac{\pi}{4}\right)$
78.	$(6.115, 1.561, 4.052)$	$(6.311, 0.25, 4.052)$	$(7.5, 0.25, 1)$
79.	$(3, -2, 2)$	$(3.606, -0.588, 2)$	$(4.123, -0.588, 1.064)$
80.	$(3\sqrt{2}, 3\sqrt{2}, -3)$	$(6, 0.785, -3)$	$(6.708, 0.785, 2.034)$
81.	$\left(\frac{5}{2}, \frac{4}{3}, \frac{-3}{2}\right)$	$(2.833, 0.490, -1.5)$	$(3.206, 0.490, 2.058)$
82.	$(0, -5, 4)$	$(5, -1.571, 4)$	$(6.403, -1.571, 0.896)$
83.	$(-3.536, 3.536, -5)$	$\left(5, \frac{3\pi}{4}, -5\right)$	$(7.071, 2.356, 2.356)$
84.	$(-1.732, 1, 3)$	$\left(-2, \frac{11\pi}{6}, 3\right)$	$(3.606, 2.618, 0.588)$

[**Note:** Use the cylindrical coordinate $\left(2, \frac{5\pi}{6}, 3\right)$]

85.	$(2.804, -2.095, 6)$	$(-3.5, 2.5, 6)$	$(6.946, 5.642, 0.528)$

[**Note:** Use the cylindrical coordinates $(3.5, 5.642, 6)$]

86.	$(2.207, 7.949, -4)$	$(8.25, 1.3, -4)$	$(9.169, 1.3, 2.022)$

87. $r = 5$

Cylinder

Matches graph (d)

88. $\theta = \frac{\pi}{4}$

Plane

Matches graph (e)

89. $\rho = 5$

Sphere

Matches graph (c)

90. $\phi = \frac{\pi}{4}$

Cone

Matches graph (a)

91. $r^2 = z, x^2 + y^2 = z$

Paraboloid

Matches graph (f)

92. $\rho = 4 \sec \phi, z = \rho \cos \phi = 4$

Plane

Matches graph (b)

93. Rectangular to cylindrical: $r^2 = x^2 + y^2$

$\tan \theta = \frac{y}{x}$

$z = z$

Cylindrical to rectangular: $x = r \cos \theta$

$y = r \sin \theta$

$z = z$

94. $r = a$ Cylinder with z-axis symmetry

$\theta = b$ Plane perpendicular to xy-plane

$z = c$ Plane parallel to xy-plane

95. Rectangular to spherical: $\rho^2 = x^2 + y^2 + z^2$

$\tan \theta = \frac{y}{x}$

$\phi = \arccos\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right)$

Spherical to rectangular: $x = \rho \sin \phi \cos \theta$

$y = \rho \sin \phi \sin \theta$

$z = \rho \cos \phi$

96. $\rho = a$ Sphere

$\theta = b$ Vertical half-plane

$\phi = c$ Half-cone

97. $x^2 + y^2 + z^2 = 16$

(a) $r^2 + z^2 = 16$

(b) $\rho^2 = 16, \rho = 4$

98. $4(x^2 + y^2) = z^2$

(a) $4r^2 = z^2, 2r = z$

(b) $4(\rho^2 \sin^2 \phi \cos^2 \theta + \rho^2 \sin^2 \phi \sin^2 \theta) = \rho^2 \cos^2 \phi,$

$4 \sin^2 \phi = \cos^2 \phi, \tan^2 \phi = \frac{1}{4},$

$\tan \phi = \frac{1}{2}, \phi = \arctan \frac{1}{2}$

99. $x^2 + y^2 + z^2 - 2z = 0$

(a) $r^2 + z^2 - 2z = 0, r^2 + (z - 1)^2 = 1$

(b) $\rho^2 - 2\rho \cos \phi = 0, \rho(\rho - 2 \cos \phi) = 0,$

$\rho = 2 \cos \phi$

100. $x^2 + y^2 = z$

(a) $r^2 = z$

(b) $\rho^2 \sin^2 \phi = \rho \cos \phi, \rho \sin^2 \phi = \cos \phi,$

$\rho = \frac{\cos \phi}{\sin^2 \phi}, \rho = \csc \phi \cot \phi$

101. $x^2 + y^2 = 4y$

(a) $r^2 = 4r \sin \theta,\ r = 4 \sin \theta$

(b) $\rho^2 \sin^2 \phi = 4\rho \sin \phi \sin \theta,$

$\rho \sin \phi(\rho \sin \phi - 4 \sin \theta) = 0,$

$\rho = \frac{4 \sin \theta}{\sin \phi},\ \rho = 4 \sin \theta \csc \phi$

102. $x^2 + y^2 = 16$

(a) $r^2 = 16, r = 4$

(b) $\rho^2 \sin^2 \phi = 16, \rho^2 \sin^2 \phi - 16 = 0,$

$(\rho \sin \phi - 4)(\rho \sin \phi + 4) = 0, \rho = 4 \csc \phi$

103. $x^2 - y^2 = 9$

(a) $r^2\cos^2\theta - r^2\sin^2\theta = 9$,

$$r^2 = \frac{9}{\cos^2\theta - \sin^2\theta}$$

(b) $\rho^2\sin^2\phi\cos^2\theta - \rho^2\sin^2\phi\sin^2\theta = 9$,

$$\rho^2\sin^2\phi = \frac{9}{\cos^2\theta - \sin^2\theta},$$

$$\rho^2 = \frac{9\csc^2\phi}{\cos^2\theta - \sin^2\theta}$$

104. $y = 4$

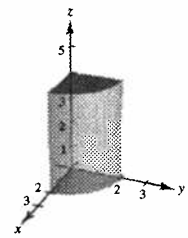
(a) $r\sin\theta = 4,\ r = 4\csc\theta$

(b) $\rho\sin\phi\sin\theta = 4,\ \rho = 4\csc\phi\csc\theta$

105. $0 \le \theta \le \dfrac{\pi}{2}$

$0 \le r \le 2$

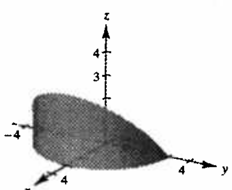
$0 \le z \le 4$



106. $-\dfrac{\pi}{2} \le \theta \le \dfrac{\pi}{2}$

$0 \le r \le 3$

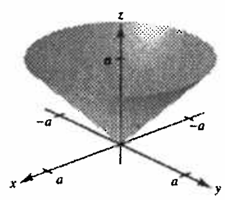
$0 \le z \le r\cos\theta$



107. $0 \le \theta \le 2\pi$

$0 \le r \le a$

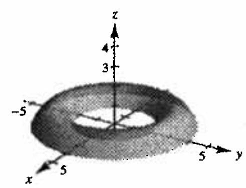
$r \le z \le a$



108. $0 \le \theta \le 2\pi$

$2 \le r \le 4$

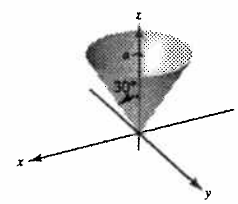
$z^2 \le -r^2 + 6r - 8$



109. $0 \le \theta \le 2\pi$

$0 \le \phi \le \dfrac{\pi}{6}$

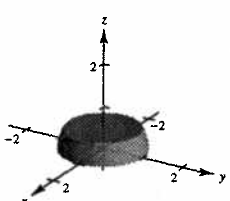
$0 \le \rho \le a\sec\phi$



110. $0 \le \theta \le 2\pi$

$\dfrac{\pi}{4} \le \phi \le \dfrac{\pi}{2}$

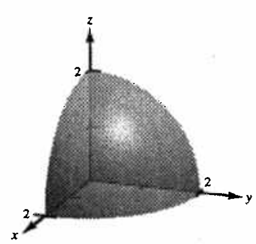
$0 \le \rho \le 1$



111. $0 \le \theta \le \dfrac{\pi}{2}$

$0 \le \phi \le \dfrac{\pi}{2}$

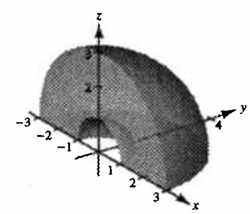
$0 \le \rho \le 2$



112. $0 \le \theta \le \pi$

$0 \le \phi \le \dfrac{\pi}{2}$

$1 \le \rho \le 3$

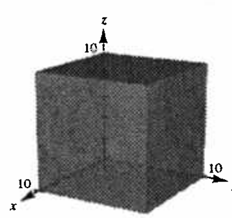


113. Rectangular

$0 \le x \le 10$

$0 \le y \le 10$

$0 \le z \le 10$



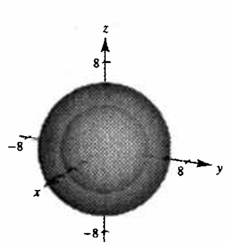
114. Cylindrical:

$0.75 \le r \le 1.25$

$0 \le z \le 8$

115. Spherical

$4 \le \rho \le 6$

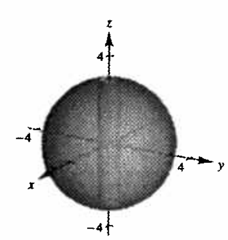


116. Cylindrical

$\frac{1}{2} \le r \le 3$

$0 \le \theta \le 2\pi$

$-\sqrt{9 - r^2} \le z \le \sqrt{9 - r^2}$



117. Cylindrical coordinates:

$r^2 + z^2 \le 9,$

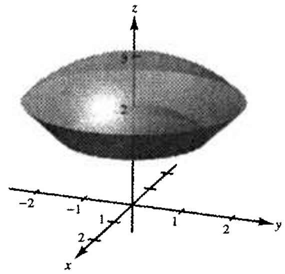
$r \le 3\cos\theta, 0 \le \theta \le 2\pi$

118. Spherical coordinates:

$\rho \ge 2$

$\rho \le 3$

$0 \le \phi \le \frac{\pi}{4}$



119. False. $\theta = c$ represents a half-plane.

120. True. They both represent spheres of radius 2 centered at the origin.

121. False. $(r, \theta, z) = (0, 0, 1)$ and $(r, \theta, z) = (0, \pi, 1)$ represent the same point $(x, y, z) = (0, 0, 1)$.

122. True (except for the orgin).

123. $z = \sin\theta, r = 1$

$z = \sin\theta = \frac{y}{r} = \frac{y}{1} = y$

The curve of intersection is the ellipse formed by the intersection of the plane $z = y$ and the cylinder $r = 1$.

124. $\rho = 2\sec\phi \Rightarrow \rho\cos\phi = 2 \Rightarrow z = 2$ plane

$\rho = 4$ sphere

The intersection of the plane and the sphere is a circle.

Review Exercises for Chapter 11

1. $P = (1, 2),\ Q = (4, 1),\ R = (5, 4)$

(a) $\mathbf{u} = \overrightarrow{PQ} = \langle 3, -1\rangle = 3\mathbf{i} - \mathbf{j},$

$\mathbf{v} = \overrightarrow{PR} = \langle 4, 2\rangle = 4\mathbf{i} + 2\mathbf{j}$

(b) $\|\mathbf{v}\| = \sqrt{4^2 + 2^2} = 2\sqrt{5}$

(c) $2\mathbf{u} + \mathbf{v} = \langle 6, -2\rangle + \langle 4, 2\rangle = \langle 10, 0\rangle = 10\mathbf{i}$

2. $P = (-2, -1),\ Q = (5, -1)\ R = (2, 4)$

(a) $\mathbf{u} = \overrightarrow{PQ} = \langle 7, 0\rangle = 7\mathbf{i},\ \mathbf{v} = \overrightarrow{PR} = \langle 4, 5\rangle = 4\mathbf{i} + 5\mathbf{j}$

(b) $\|\mathbf{v}\| = \sqrt{4^2 + 5^2} = \sqrt{41}$

(c) $2\mathbf{u} + \mathbf{v} = 14\mathbf{i} + (4\mathbf{i} + 5\mathbf{j}) = 18\mathbf{i} + 5\mathbf{j}$

3. $\mathbf{v} = \|\mathbf{v}\|\cos\theta\,\mathbf{i} + \|\mathbf{v}\|\sin\theta\,\mathbf{j} = 8\cos 120°\,\mathbf{i} + 8\sin 120°\,\mathbf{j}$

$= -4\mathbf{i} + 4\sqrt{3}\mathbf{j}$

4. $\mathbf{v} = \|\mathbf{v}\|\cos\theta\,\mathbf{i} + \|\mathbf{v}\|\sin\theta\,\mathbf{j} = \frac{1}{2}\cos 225°\,\mathbf{i} + \frac{1}{2}\sin 225°\,\mathbf{j}$

$= -\frac{\sqrt{2}}{4}\mathbf{i} - \frac{\sqrt{2}}{4}\mathbf{j}$

5. $z = 0,\ y = 4,\ x = -5:\ (-5, 4, 0)$

6. $x = z = 0,\ y = -7:\ (0, -7, 0)$

7. Looking down from the positive x-axis towards the yz-plane, the point is either in the first quadrant $(y > 0, z > 0)$ or in the third quadrant $(y < 0, z < 0)$. The x-coordinate can be any number.

8. Looking towards the xy-plane from the positive z-axis. The point is either in the second quadrant $(x < 0, y > 0)$ or in the fourth quadrant $(x > 0, y < 0)$. The z-coordinate can be any number.

9. $(x - 3)^2 + (y + 2)^2 + (z - 6)^2 = \left(\frac{15}{2}\right)^2$

10. Center: $\left(\frac{0 + 4}{2}, \frac{0 + 6}{2}, \frac{4 + 0}{2}\right) = (2, 3, 2)$

Radius: $\sqrt{(2 - 0)^2 + (3 - 0)^2 + (2 - 4)^2} = \sqrt{4 + 9 + 4} = \sqrt{17}$

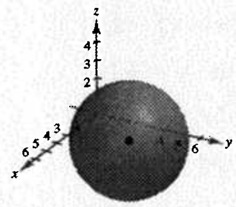
$(x - 2)^2 + (y - 3)^2 + (z - 2)^2 = 17$

11. $(x^2 - 4x + 4) + (y^2 - 6y + 9) + z^2 = -4 + 4 + 9$

$(x - 2)^2 + (y - 3)^2 + z^2 = 9$

Center: $(2, 3, 0)$

Radius: 3

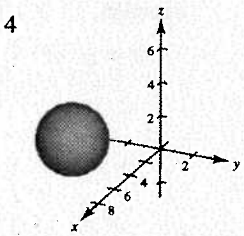


12. $(x^2 - 10x + 25) + (y^2 + 6y + 9) + (z^2 - 4z + 4) = -34 + 25 + 9 + 4$

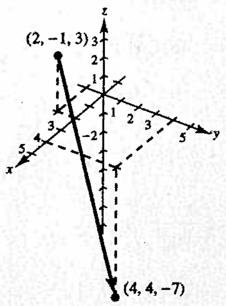
$(x - 5)^2 + (y + 3)^2 + (z - 2)^2 = 4$

Center: $(5, -3, 2)$

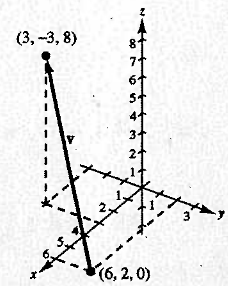
Radius: 2



13. $\mathbf{v} = \langle 4 - 2, 4 + 1, -7 - 3\rangle = \langle 2, 5, -10\rangle$



14. $\mathbf{v} = \langle 3 - 6, -3 - 2, 8 - 0\rangle = \langle -3, -5, 8\rangle$



15. $\mathbf{v} = \langle -1 - 3, 6 - 4, 9 + 1\rangle = \langle -4, 2, 10\rangle$

$\mathbf{w} = \langle 5 - 3, 3 - 4, -6 + 1\rangle = \langle 2, -1, -5\rangle$

Since $-2\mathbf{w} = \mathbf{v}$, the points lie in a straight line.

16. $\mathbf{v} = \langle 8 - 5, -5 + 4, 5 - 7\rangle = \langle 3, -1, -2\rangle$

$\mathbf{w} = \langle 11 - 5, 6 + 4, 3 - 7\rangle = \langle 6, 10, -4\rangle$

Since $\mathbf{v}$ and $\mathbf{w}$ are not parallel, the points do not lie in a straight line.

17. Unit vector: $\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = \dfrac{\langle 2, 3, 5\rangle}{\sqrt{38}} = \left\langle \dfrac{2}{\sqrt{38}}, \dfrac{3}{\sqrt{38}}, \dfrac{5}{\sqrt{38}}\right\rangle$

18. $8\,\dfrac{\langle 6, -3, 2\rangle}{\sqrt{49}} = \dfrac{8}{7}\langle 6, -3, 2\rangle = \left\langle \dfrac{48}{7}, -\dfrac{24}{7}, \dfrac{16}{7}\right\rangle$

19. $P = (5, 0, 0)$, $Q = (4, 4, 0)$, $R = (2, 0, 6)$

(a) $\mathbf{u} = \overrightarrow{PQ} = \langle -1, 4, 0\rangle = -\mathbf{i} + 4\mathbf{j}$,

$\mathbf{v} = \overrightarrow{PR} = \langle -3, 0, 6\rangle = -3\mathbf{i} + 6\mathbf{k}$

(b) $\mathbf{u} \cdot \mathbf{v} = (-1)(-3) + 4(0) + 0(6) = 3$

(c) $\mathbf{v} \cdot \mathbf{v} = 9 + 36 = 45$

20. $P = (2, -1, 3)$, $Q = (0, 5, 1)$, $R = (5, 5, 0)$

(a) $\mathbf{u} = \overrightarrow{PQ} = \langle -2, 6, -2\rangle = -2\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$,

$\mathbf{v} = \overrightarrow{PR} = \langle 3, 6, -3\rangle = 3\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}$

(b) $\mathbf{u} \cdot \mathbf{v} = (-2)(3) + (6)(6) + (-2)(-3) = 36$

(c) $\mathbf{v} \cdot \mathbf{v} = 9 + 36 + 9 = 54$

21. $\mathbf{u} = \langle 7, -2, 3\rangle$, $\mathbf{v} = \langle -1, 4, 5\rangle$

Since $\mathbf{u} \cdot \mathbf{v} = 0$, the vectors are orthogonal.

22. $\mathbf{u} = \langle -4, 3, -6\rangle$, $\mathbf{v} = \langle 16, -12, 24\rangle$

Since $\mathbf{v} = -4\mathbf{u}$, the vectors are parallel.

23. $\mathbf{u} = 5\left(\cos\dfrac{3\pi}{4}\mathbf{i} + \sin\dfrac{3\pi}{4}\mathbf{j}\right) = \dfrac{5\sqrt{2}}{2}[-\mathbf{i} + \mathbf{j}]$

$\mathbf{v} = 2\left(\cos\dfrac{2\pi}{3}\mathbf{i} + \sin\dfrac{2\pi}{3}\mathbf{j}\right) = -\mathbf{i} + \sqrt{3}\mathbf{j}$

$\mathbf{u} \cdot \mathbf{v} = \dfrac{5\sqrt{2}}{2}(1 + \sqrt{3})$

$\|\mathbf{u}\| = 5 \qquad \|\mathbf{v}\| = 2$

$\cos\theta = \dfrac{|\mathbf{u} \cdot \mathbf{v}|}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \dfrac{(5\sqrt{2}/2)(1 + \sqrt{3})}{5(2)} = \dfrac{\sqrt{2} + \sqrt{6}}{4}$

$\theta = \arccos\dfrac{\sqrt{2} + \sqrt{6}}{4} = 15^\circ \left[\text{or, } \dfrac{3\pi}{4} \cdot \dfrac{2\pi}{3} = \dfrac{\pi}{12} \text{ or } 15^\circ\right]$

24. $\mathbf{u} = \langle 4, -1, 5\rangle$, $\mathbf{v} = \langle 3, 2, -2\rangle$

$\mathbf{u} \cdot \mathbf{v} = 0 \Longrightarrow$ is orthogonal to $\mathbf{v}$.

$\theta = \dfrac{\pi}{2}$

25. $\mathbf{u} = \langle 10, -5, 15\rangle$, $\mathbf{v} = \langle -2, 1, -3\rangle$

$\mathbf{u} = -5\mathbf{v} \implies \mathbf{u}$ is parallel to $\mathbf{v}$ and in the opposite direction.

$\theta = \pi$

26. $\mathbf{u} = \langle 1, 0, -3\rangle$

$\mathbf{v} = \langle 2, -2, 1\rangle$

$\mathbf{u}\cdot\mathbf{v} = -1$

$\|\mathbf{u}\| = \sqrt{10}$

$\|\mathbf{v}\| = 3$

$$\cos\theta = \frac{|\mathbf{u}\cdot\mathbf{v}|}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{1}{3\sqrt{10}}$$

$\theta \approx 83.9°$

27. There are many correct answers.
For example: $\mathbf{v} = \pm\langle 6, -5, 0\rangle$.

28. $W = \mathbf{F}\cdot\overrightarrow{PQ} = \|\mathbf{F}\|\,\|\overrightarrow{PQ}\|\cos\theta = (75)(8)\cos 30°$
$= 300\sqrt{3}$ ft · lb

In Exercises 29–38, $\mathbf{u} = \langle 3, -2, 1\rangle$, $\mathbf{v} = \langle 2, -4, -3\rangle$, $\mathbf{w} = \langle -1, 2, 2\rangle$.

29. $\mathbf{u}\cdot\mathbf{u} = 3(3) + (-2)(-2) + (1)(1)$
$= 14 = \left(\sqrt{14}\right)^2 = \|\mathbf{u}\|^2$

30. $\cos\theta = \dfrac{|\mathbf{u}\cdot\mathbf{v}|}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \dfrac{11}{\sqrt{14}\sqrt{29}}$

$\theta = \arccos\left(\dfrac{11}{\sqrt{14}\sqrt{29}}\right) \approx 56.9°$

31. $\text{proj}_{\mathbf{u}}\mathbf{w} = \left(\dfrac{\mathbf{u}\cdot\mathbf{w}}{\|\mathbf{u}\|^2}\right)\mathbf{u}$

$= -\dfrac{5}{14}\langle 3, -2, 1\rangle$

$= \left\langle -\dfrac{15}{14}, \dfrac{10}{14}, -\dfrac{5}{14}\right\rangle$

$= \left\langle -\dfrac{15}{14}, \dfrac{5}{7}, -\dfrac{5}{14}\right\rangle$

32. Work $= \mathbf{u}\cdot\mathbf{w} = -3 - 4 + 2 = -5$

33. $\mathbf{n} = \mathbf{v}\times\mathbf{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -4 & -3 \\ -1 & 2 & 2 \end{vmatrix} = -2\mathbf{i} - \mathbf{j}$

$\|\mathbf{n}\| = \sqrt{5}$

$\dfrac{\mathbf{n}}{\|\mathbf{n}\|} = \dfrac{1}{\sqrt{5}}(-2\mathbf{i} - \mathbf{j})$, unit vector or $\dfrac{1}{\sqrt{5}}(2\mathbf{i} + \mathbf{j})$

34. $\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -2 & 1 \\ 2 & -4 & -3 \end{vmatrix} = 10\mathbf{i} + 11\mathbf{j} - 8\mathbf{k}$

$\mathbf{v}\times\mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -4 & -3 \\ 3 & -2 & 1 \end{vmatrix} = -10\mathbf{i} - 11\mathbf{j} + 8\mathbf{k}$

Thus, $\mathbf{u}\times\mathbf{v} = -(\mathbf{v}\times\mathbf{u})$.

35. $V = |\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w})|$
$= |\langle 3, -2, 1\rangle\cdot\langle -2, -1, 0\rangle| = |-4| = 4$

36. $\mathbf{u}\times(\mathbf{v}+\mathbf{w}) = \langle 3, -2, 1\rangle \times \langle 1, -2, -1\rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -2 & 1 \\ 1 & -2 & -1 \end{vmatrix} = 4\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}$

$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -2 & 1 \\ 2 & -4 & -3 \end{vmatrix} = 10\mathbf{i} + 11\mathbf{j} - 8\mathbf{k}$

$\mathbf{u}\times\mathbf{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -2 & 1 \\ -1 & 2 & 2 \end{vmatrix} = -6\mathbf{i} - 7\mathbf{j} + 4\mathbf{k}$

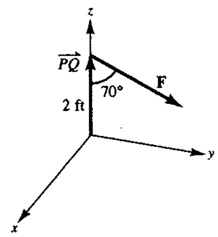
$(\mathbf{u}\times\mathbf{v}) + (\mathbf{u}\times\mathbf{w}) = 4\mathbf{i} + 4\mathbf{j} - 4\mathbf{k} = \mathbf{u}\times(\mathbf{v}+\mathbf{w})$

37. Area parallelogram $= \|\mathbf{u} \times \mathbf{v}\| = \|\langle 10, 11, -8\rangle\| = \sqrt{10^2 + 11^2 + (-8)^2}$ (See Exercises 34, 36)

$= \sqrt{285}$

38. Area triangle $= \frac{1}{2}\|\mathbf{v} \times \mathbf{w}\| = \frac{1}{2}\sqrt{(-2)^2 + (-1)^2} = \frac{\sqrt{5}}{2}$ (See Exercise 33)

39. $\mathbf{F} = c(\cos 20°\mathbf{j} + \sin 20°\mathbf{k})$



$\overrightarrow{PQ} = 2\mathbf{k}$

$$\overrightarrow{PQ} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2 \\ 0 & c\cos 20° & c\sin 20° \end{vmatrix} = -2c\cos 20°\mathbf{i}$$

$200 = \|\overrightarrow{PQ} \times \mathbf{F}\| = 2c\cos 20°$

$c = \dfrac{100}{\cos 20°}$

$\mathbf{F} = \dfrac{100}{\cos 20°}(\cos 20°\mathbf{j} + \sin 20°\mathbf{k}) = 100(\mathbf{j} + \tan 20°\mathbf{k})$

$\|\mathbf{F}\| = 100\sqrt{1 + \tan^2 20°} = 100 \sec 20° \approx 106.4$ lb

40. $V = |\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})| = \begin{vmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & -1 & 2 \end{vmatrix} = 2(5) = 10$

41. $\mathbf{v} = \langle 9 - 3, 11 - 0, 6 - 2\rangle = \langle 6, 11, 4\rangle$

(a) Parametric equations: $x = 3 + 6t, y = 11t, z = 2 + 4t$

(b) Symmetric equations: $\dfrac{x - 3}{6} = \dfrac{y}{11} = \dfrac{z - 2}{4}$

42. $\mathbf{v} = \langle 8 + 1, 10 - 4, 5 - 3\rangle = \langle 9, 6, 2\rangle$

(a) Parametric equations: $x = -1 + 9t, y = 4 + 6t, z = 3 + 2t$

(b) Symmetric equations: $\dfrac{x + 1}{9} = \dfrac{y - 4}{6} = \dfrac{z - 3}{2}$

43. $\mathbf{v} = \mathbf{j}$

(a) $x = 1,\ y = 2 + t,\ z = 3$

(b) None

44. Direction numbers: 1, 1, 1

(a) $x = 1 + t,\ y = 2 + t,\ z = 3 + t$

(b) $x - 1 = y - 2 = z - 3$

45. $3x - 3y - 7z = -4,\ x - y + 2z = 3$

Solving simultaneously, we have $z = 1$. Substituting $z = 1$ into the second equation we have $y = x - 1$. Substituting for x in this equation we obtain two points on the line of intersection, $(0, -1, 1), (1, 0, 1)$. The direction vector of the line of intersection is $\mathbf{v} = \mathbf{i} + \mathbf{j}$.

(a) $x = t,\ y = -1 + t,\ z = 1$

(b) $x = y + 1,\ z = 1$

46. $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -5 & 1 \\ -3 & 1 & 4 \end{vmatrix} = -21\mathbf{i} - 11\mathbf{j} - 13\mathbf{k}$

Direction numbers: 21, 11, 13

(a) $x = 21t,\ y = 1 + 11t,\ z = 4 + 13t$

(b) $\dfrac{x}{21} = \dfrac{y - 1}{11} = \dfrac{z - 4}{13}$

47. $P = (-3, -4, 2),\ Q = (-3, 4, 1),\ R = (1, 1, -2)$

$\overrightarrow{PQ} = \langle 0, 8, -1\rangle,\ \overrightarrow{PR} = \langle 4, 5, -4\rangle$

$$\mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 8 & -1 \\ 4 & 5 & -4 \end{vmatrix} = -27\mathbf{i} - 4\mathbf{j} - 32\mathbf{k}$$

$$-27(x + 3) - 4(y + 4) - 32(z - 2) = 0$$

$$27x + 4y + 32z = -33$$

48. $\mathbf{n} = 3\mathbf{i} - \mathbf{j} + \mathbf{k}$

$$3(x + 2) - 1(y - 3) + 1(z - 1) = 0$$

$$3x - y + z + 8 = 0$$

49. The two lines are parallel as they have the same direction numbers, $-2, 1, 1$. Therefore, a vector parallel to the plane is $\mathbf{v} = -2\mathbf{i} + \mathbf{j} + \mathbf{k}$. A point on the first line is $(1, 0, -1)$ and a point on the second line is $(-1, 1, 2)$. The vector $\mathbf{u} = 2\mathbf{i} - \mathbf{j} - 3\mathbf{k}$ connecting these two points is also parallel to the plane. Therefore, a normal to the plane is

$$\mathbf{v} \times \mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 1 & 1 \\ 2 & -1 & -3 \end{vmatrix}$$

$$= -2\mathbf{i} - 4\mathbf{j} = -2(\mathbf{i} + 2\mathbf{j}).$$

Equation of the plane: $(x - 1) + 2y = 0$

$$x + 2y = 1$$

50. Let $\mathbf{v} = \langle 5 - 2, 1 + 2, 3 - 1\rangle = \langle 3, 3, 2\rangle$ be the direction vector for the line through the two points. Let $\mathbf{n} = \langle 2, 1, -1\rangle$ be the normal vector to the plane. Then

$$\mathbf{v} \times \mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 3 & 2 \\ 2 & 1 & -1 \end{vmatrix} = \langle -5, 7, -3\rangle$$

is the normal to the unknown plane.

$$-5(x - 5) + 7(y - 1) - 3(z - 3) = 0$$

$$-5x + 7y - 3z + 27 = 0$$

51. $Q(1, 0, 2)$ point

$2x - 3y + 6z = 6$

A point P on the plane is $(3, 0, 0)$.

$\overrightarrow{PQ} = \langle -2, 0, 2\rangle$

$\mathbf{n} = \langle 2, -3, 6\rangle$ normal to plane

$$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \frac{8}{7}$$

52. $Q(3, -2, 4)$ point

$P(5, 0, 0)$ point on plane

$\mathbf{n} = \langle 2, -5, 1\rangle$ normal to plane

$\overrightarrow{PQ} = \langle -2, -2, 4\rangle$

$$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \frac{10}{\sqrt{30}} = \frac{\sqrt{30}}{3}$$

53. The normal vectors to the planes are the same,

$\mathbf{n} = \langle 5, -3, 1\rangle$.

Choose a point in the first plane, $P = (0, 0, 2)$. Choose a point in the second plane, $Q = (0, 0, -3)$.

$\overrightarrow{PQ} = \langle 0, 0, -5\rangle$

$$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \frac{|-5|}{\sqrt{35}} = \frac{5}{\sqrt{35}} = \frac{\sqrt{35}}{7}$$

54. $Q(-5, 1, 3)$ point

$\mathbf{u} = \langle 1, -2, -1\rangle$ direction vector

$P = (1, 3, 5)$ point on line

$\overrightarrow{PQ} = \langle -6, -2, -2\rangle$

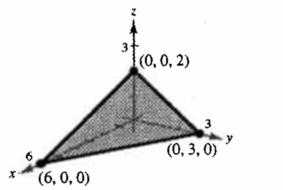
$$\overrightarrow{PQ} \times \mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -6 & -2 & -2 \\ 1 & -2 & -1 \end{vmatrix} = \langle -2, -8, 14\rangle$$

$$D = \frac{\|\overrightarrow{PQ} \times \mathbf{u}\|}{\|\mathbf{u}\|} = \frac{\sqrt{264}}{\sqrt{6}} = 2\sqrt{11}$$

55. $x + 2y + 3z = 6$

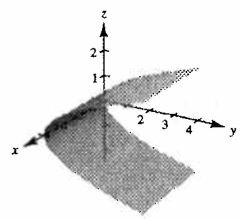
Plane

Intercepts: $(6, 0, 0)$, $(0, 3, 0)$, $(0, 0, 2)$



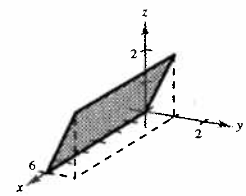
56. $y = z^2$

Since the x-coordinate is missing, we have a cylindrical surface with rulings parallel to the x-axis. The generating curve is a parabola in the yz-coordinate plane.



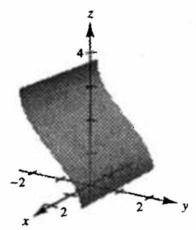
57. $y = \frac{1}{2}z$

Plane with rulings parallel to the x-axis



58. $y = \cos z$

Since the x-coordinate is missing, we have a cylindrical surface with rulings parallel to the x-axis. The generating curve is $y = \cos z$.



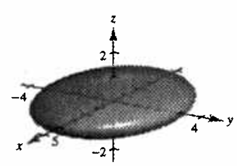
59. $\frac{x^2}{16} + \frac{y^2}{9} + z^2 = 1$

Ellipsoid

xy-trace: $\frac{x^2}{16} + \frac{y^2}{9} = 1$

xz-trace: $\frac{x^2}{16} + z^2 = 1$

yz-trace: $\frac{y^2}{9} + z^2 = 1$



60. $16x^2 + 16y^2 - 9z^2 = 0$

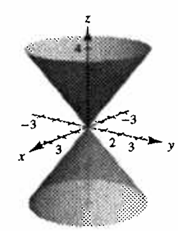
Cone

xy-trace: point $(0,0, 0)$

xz-trace: $z = \pm\frac{4x}{3}$

yz-trace: $z = \pm\frac{4y}{3}$

$z = 4,\ x^2 + y^2 = 9$



61. $\frac{x^2}{16} - \frac{y^2}{9} + z^2 = -1$

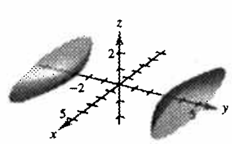
$\frac{y^2}{9} - \frac{x^2}{16} - z^2 = 1$

Hyperboloid of two sheets

xy-trace: $\frac{y^2}{9} - \frac{x^2}{16} = 1$

xz-trace: None

yz-trace: $\frac{y^2}{9} - z^2 = 1$



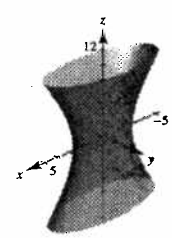
62. $\frac{x^2}{25} + \frac{y^2}{4} - \frac{z^2}{100} = 1$

Hyperboloid of one sheet

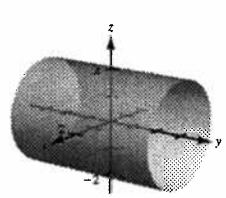
xy-trace: $\frac{x^2}{25} + \frac{y^2}{4} = 1$

xz-trace: $\frac{x^2}{25} - \frac{z^2}{100} = 1$

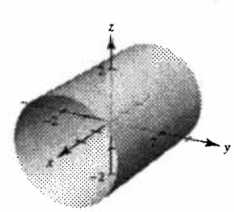
yz-trace: $\frac{y^2}{4} - \frac{z^2}{100} = 1$



63. $x^2 + z^2 = 4$. Cylinder of radius 2 about y-axis



64. $y^2 + z^2 = 16$. Cylinder of radius 4 about x-axis



65. Let $y = r(x) = 2\sqrt{x}$ and revolve the curve about the x-axis.

66. $z^2 = 2y$ revolved about y-axis

$z = \pm\sqrt{2y}$

$x^2 + z^2 = [r(y)]^2 = 2y$

$x^2 + z^2 = 2y$

67. $\left(-2\sqrt{2}, 2\sqrt{2}, 2\right)$, rectangular

(a) $r = \sqrt{\left(-2\sqrt{2}\right)^2 + \left(2\sqrt{2}\right)^2} = 4,\ \theta = \arctan(-1) = \frac{3\pi}{4},\ z = 2,\ \left(4, \frac{3\pi}{4}, 2\right)$, cylindrical

(b) $\rho = \sqrt{\left(-2\sqrt{2}\right)^2 + \left(2\sqrt{2}\right)^2 + (2)^2} = 2\sqrt{5},\ \theta = \frac{3\pi}{4},\ \phi = \arccos\frac{2}{2\sqrt{5}} = \arccos\frac{1}{\sqrt{5}},\ \left(2\sqrt{5}, \frac{3\pi}{4}, \arccos\frac{\sqrt{5}}{5}\right)$, spherical

68. $\left(\frac{\sqrt{3}}{4}, \frac{3}{4}, \frac{3\sqrt{3}}{2}\right)$, rectangular

(a) $r = \sqrt{\left(\frac{\sqrt{3}}{4}\right)^2 + \left(\frac{3}{4}\right)^2} = \frac{\sqrt{3}}{2},\ \theta = \arctan\sqrt{3} = \frac{\pi}{3},\ z = \frac{3\sqrt{3}}{2},\ \left(\frac{\sqrt{3}}{2}, \frac{\pi}{2}, \frac{3\sqrt{3}}{2}\right)$, cylindrical

(b) $\rho = \sqrt{\left(\frac{\sqrt{3}}{4}\right)^2 + \left(\frac{3}{4}\right)^2 + \left(\frac{3\sqrt{3}}{2}\right)^2} = \frac{\sqrt{30}}{2},\ \theta = \frac{\pi}{3},\ \phi = \arccos\frac{3}{\sqrt{10}},\ \left(\frac{\sqrt{30}}{2}, \frac{\pi}{3}, \arccos\frac{3}{\sqrt{10}}\right)$, spherical

69. $\left(100, -\frac{\pi}{6}, 50\right)$, cylindrical

$\rho = \sqrt{100^2 + 50^2} = 50\sqrt{5}$

$\theta = -\frac{\pi}{6}$

$\phi = \arccos\left(\frac{50}{50\sqrt{5}}\right) = \arccos\left(\frac{1}{\sqrt{5}}\right) \approx 63.4°$ or 1.107

$\left(50\sqrt{5}, -\frac{\pi}{6}, 63.4°\right)$, spherical or $\left(50\sqrt{5}, \frac{-\pi}{6}, 1.1071\right)$

70. $\left(81, -\frac{5\pi}{6}, 27\sqrt{3}\right)$, cylindrical

$\rho = \sqrt{6561 + 2187} = 54\sqrt{3}$

$\theta = -\frac{5\pi}{6}$

$\phi = \arccos\left(\frac{27\sqrt{3}}{54\sqrt{3}}\right) = \arccos\frac{1}{2} = \frac{\pi}{3}$

$\left(54\sqrt{3}, -\frac{5\pi}{6}, \frac{\pi}{3}\right)$, spherical

71. $\left(25, -\frac{\pi}{4}, \frac{3\pi}{4}\right)$, spherical

$r^2 = \left(25\sin\left(\frac{3\pi}{4}\right)\right)^2 \implies r = 25\frac{\sqrt{2}}{2}$

$\theta = -\frac{\pi}{4}$

$z = \rho\cos\phi = 25\cos\frac{3\pi}{4} = -25\frac{\sqrt{2}}{2}$

$\left(25\frac{\sqrt{2}}{2}, -\frac{\pi}{4}, -\frac{25\sqrt{2}}{2}\right)$, cylindrical

72. $\left(12, -\frac{\pi}{2}, \frac{2\pi}{3}\right)$, spherical

$r^2 = \left(12\sin\left(\frac{2\pi}{3}\right)\right)^2 \implies r = 6\sqrt{3}$

$\theta = -\frac{\pi}{2}$

$z = \rho\cos\phi = 12\cos\left(\frac{2\pi}{3}\right) = -6$

$\left(6\sqrt{3}, -\frac{\pi}{2}, -6\right)$, cylindrical

73. $x^2 - y^2 = 2z$

(a) Cylindrical: $r^2\cos^2\theta - r^2\sin^2\theta = 2z,\ r^2\cos 2\theta = 2z$

(b) Spherical: $\rho^2\sin^2\phi\cos^2\theta - \rho^2\sin^2\phi\sin^2\theta = 2\rho\cos\phi,\ \rho\sin^2\phi\cos 2\theta - 2\cos\phi = 0,\ \rho = 2\sec 2\theta\cos\phi\csc^2\phi$

74. $x^2 + y^2 + z^2 = 16$

(a) Cylindrical: $r^2 + z^2 = 16$

(b) Spherical: $\rho = 4$

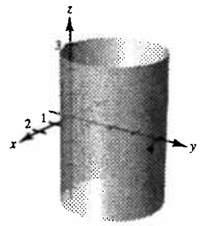
75. $r = 4 \sin \theta$ cylindrical coordinates

$r^2 = 4r \sin \theta$

$x^2 + y^2 = 4y$

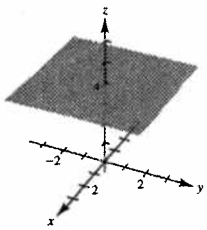
$x^2 + y^2 - 4y + 4 = 4$

$x^2 + (y - 2)^2 = 4$ rectangular coordinates



76. $z = 4$ cylindrical coordinates

$z = 4$ rectangular coordinates

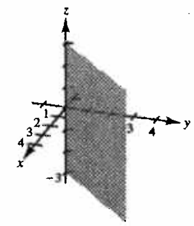


77. $\theta = \dfrac{\pi}{4}$ spherical coordinates

$\tan \theta = \tan \dfrac{\pi}{4} = 1$

$\dfrac{y}{x} = 1$

$y = x, x \geq 0$ rectangular coordinates half-plane



78. $\rho = 2 \cos \theta$ spherical coordinates

Because $\cos \theta = \dfrac{x}{r} = \dfrac{x}{\sqrt{x^2 + y^2}}$, we have

$\sqrt{x^2 + y^2 + z^2} = 2\dfrac{x}{\sqrt{x^2 + y^2}}$ rectangular equation

Problem Solving for Chapter 11

1. $\mathbf{a} + \mathbf{b} + \mathbf{c} = \mathbf{0}$

$\mathbf{b} \times (\mathbf{a} + \mathbf{b} + \mathbf{c}) = \mathbf{0}$

$(\mathbf{b} \times \mathbf{a}) + (\mathbf{b} \times \mathbf{c}) = \mathbf{0}$

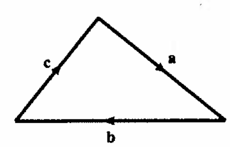
$\|\mathbf{a} \times \mathbf{b}\| = \|\mathbf{b} \times \mathbf{c}\|$

$\|\mathbf{b} \times \mathbf{c}\| = \|\mathbf{b}\| \|\mathbf{c}\| \sin A$

$\|\mathbf{a} \times \mathbf{b}\| = \|\mathbf{a}\| \|\mathbf{b}\| \sin C$

Then,

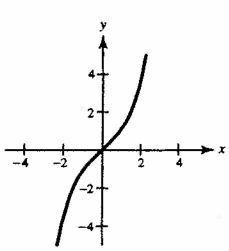
$$\frac{\sin A}{\|\mathbf{a}\|} = \frac{\|\mathbf{b} \times \mathbf{c}\|}{\|\mathbf{a}\| \|\mathbf{b}\| \|\mathbf{c}\|} = \frac{\|\mathbf{a} \times \mathbf{b}\|}{\|\mathbf{a}\| \|\mathbf{b}\| \|\mathbf{c}\|} = \frac{\sin C}{\|\mathbf{c}\|}.$$



The other case, $\dfrac{\sin A}{\|\mathbf{a}\|} = \dfrac{\sin B}{\|\mathbf{b}\|}$ is similar.

2. $f(x) = \displaystyle\int_0^x \sqrt{t^4 + 1}\, dt$

(a)



(b) $f'(x) = \sqrt{x^4 + 1}$

$f'(0) = 1 = \tan \theta$

$\theta = \dfrac{\pi}{4}$

$\mathbf{u} = \dfrac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j}) = \left\langle \dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2} \right\rangle$

(c) $\pm\left\langle \dfrac{\sqrt{2}}{2}, -\dfrac{\sqrt{2}}{2} \right\rangle$

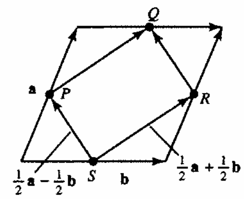
(d) The line is $y = x$: $x = t, y = t$.

3. Label the figure as indicated.

From the figure, you see that

$$\overrightarrow{SP} = \frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{b} = \overrightarrow{RQ} \quad \text{and} \quad \overrightarrow{SR} = \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b} = \overrightarrow{PQ}.$$

Since $\overrightarrow{SP} = \overrightarrow{RQ}$ and $\overrightarrow{SR} = \overrightarrow{PQ}$, $PSRQ$ is a parallelogram.



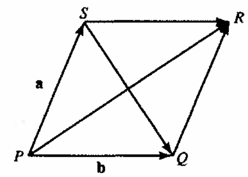
4. Label the figure as indicated.

$\overrightarrow{PR} = \mathbf{a} + \mathbf{b}$

$\overrightarrow{SQ} = \mathbf{b} - \mathbf{a}$

$(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{b} - \mathbf{a}) = \|\mathbf{b}\|^2 - \|\mathbf{a}\|^2 = 0$, because

$\|\mathbf{a}\| = \|\mathbf{b}\|$ in a rhombus.

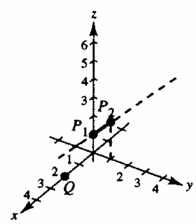


5. (a) $\mathbf{u} = \langle 0, 1, 1\rangle$ direction vector of line determined by P_1 and P_2.

$$D = \frac{\|\overrightarrow{P_1Q} \times \mathbf{u}\|}{\|\mathbf{u}\|}$$

$$= \frac{\|\langle 2, 0, -1\rangle \times \langle 0, 1, 1\rangle\|}{\sqrt{2}}$$

$$= \frac{\|\langle 1, -2, 2\rangle\|}{\sqrt{2}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$$



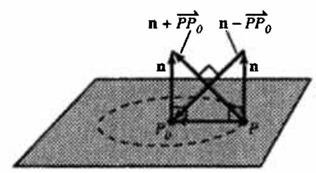
(b) The shortest distance to the line **segment** is

$\|P_1Q\| = \|\langle 2, 0, -1\rangle\| = \sqrt{5}$.

6. $(\mathbf{n} + \overrightarrow{PP_0}) \perp (\mathbf{n} - \overrightarrow{PP_0})$

Figure is a square.

Thus, $\|\overrightarrow{PP_0}\| = \|\mathbf{n}\|$ and the points P form a circle of radius $\|\mathbf{n}\|$ in the plane with center at P_0.



7. (a) $V = \pi\int_0^1 \left(\sqrt{z}\right)^2 dz = \left[\pi\frac{z^2}{2}\right]_0^1 = \frac{1}{2}\pi$

Note: $\frac{1}{2}$(base)(altitude) $= \frac{1}{2}\pi(1) = \frac{1}{2}\pi$

(b) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = z$: (slice at $z = c$)

$$\frac{x^2}{\left(\sqrt{c}a\right)^2} + \frac{y^2}{\left(\sqrt{c}b\right)^2} = 1$$

At $z = c$, figure is ellipse of area

$$\pi\left(\sqrt{c}a\right)\left(\sqrt{c}b\right) = \pi abc.$$

$$V = \int_0^k \pi abc \cdot dc = \left[\frac{\pi abc^2}{2}\right]_0^k = \frac{\pi abk^2}{2}$$

(c) $V = \frac{1}{2}(\pi abk)k = \frac{1}{2}$(area of base)(height)

8. (a) $V = 2\int_0^r \pi(r^2 - x^2)\,dx = 2\pi\left[r^2x - \frac{x^3}{3}\right]_0^r = \frac{4}{3}\pi r^3$

(b) At height $z = d > 0$,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{d^2}{c^2} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 - \frac{d^2}{c^2} = \frac{c^2 - d^2}{c^2}$$

$$\frac{x^2}{\frac{a^2(c^2 - d^2)}{c^2}} + \frac{y^2}{\frac{b^2(c^2 - d^2)}{c^2}} = 1.$$

$$\text{Area} = \pi\sqrt{\left(\frac{a^2(c^2 - d^2)}{c^2}\right)\left(\frac{b^2(c^2 - d^2)}{c^2}\right)}$$

$$= \frac{\pi ab}{c^2}(c^2 - d^2)$$

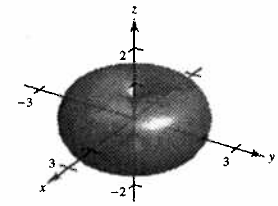
$$V = 2\int_0^c \frac{\pi ab}{c^2}(c^2 - d^2)\,dd$$

$$= \frac{2\pi ab}{c^2}\left[c^2d - \frac{d^3}{3}\right]_0^c$$

$$= \frac{4}{3}\pi abc$$

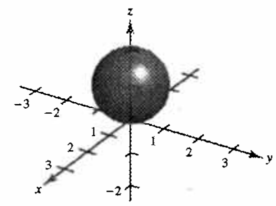
9. (a) $\rho = 2\sin\phi$

Torus



(b) $\rho = 2\cos\phi$

Sphere



10. (a) $r = 2\cos\theta$

Cylinder

(b) $z = r^2\cos 2\theta$

$z^2 = x^2 - y^2$

Hyperbolic paraboloid

11. From Exercise 64, Section 11.4,

$(\mathbf{u}\times\mathbf{v})\times(\mathbf{w}\times\mathbf{z}) = [(\mathbf{u}\times\mathbf{v})\cdot\mathbf{z}]\mathbf{w} - [(\mathbf{u}\times\mathbf{v})\cdot\mathbf{w}]\mathbf{z}.$

12. $x = -t + 3, y = \frac{1}{2}t + 1, z = 2t - 1; Q = (4, 3, s)$

(a) $\mathbf{u} = \langle -2, 1, 4\rangle$ direction vector for line

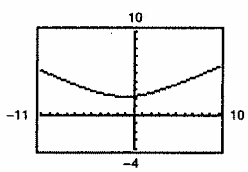
$P = (3, 1, -1)$ point on line

$\overrightarrow{PQ} = \langle 1, 2, s+1\rangle$

$$\overrightarrow{PQ}\times\mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & s+1 \\ -2 & 1 & 4 \end{vmatrix} = (7-s)\mathbf{i} + (-6-2s)\mathbf{j} + 5\mathbf{k}$$

$$D = \frac{\|\overrightarrow{PQ}\times\mathbf{u}\|}{\|\mathbf{u}\|} = \frac{\sqrt{(7-s)^2 + (-6-2s)^2 + 25}}{\sqrt{21}}$$

(b)



The minimum is $D \approx 2.2361$ at $s = -1$.

(c) Yes, there are slant asymptotes. Using $s = x$, we have

$$D(s) = \frac{1}{\sqrt{21}}\sqrt{5x^2 + 10x + 110} = \frac{\sqrt{5}}{\sqrt{21}}\sqrt{x^2 + 2x + 22}$$

$$= \frac{\sqrt{5}}{\sqrt{21}}\sqrt{(x+1)^2 + 21} \to \pm\sqrt{\frac{5}{21}}(x+1)$$

$$y = \pm\frac{\sqrt{105}}{21}(s+1) \text{ slant asymptotes.}$$

13. (a) $\mathbf{u} = \|\mathbf{u}\|(\cos 0\,\mathbf{i} + \sin 0\,\mathbf{j}) = \|\mathbf{u}\|\mathbf{i}$

Downward force $\mathbf{w} = -\mathbf{j}$

$\mathbf{T} = \|\mathbf{T}\|(\cos(90° + \theta)\mathbf{i} + \sin(90° + \theta)\mathbf{j})$

$= \|\mathbf{T}\|(-\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j})$

$\mathbf{0} = \mathbf{u} + \mathbf{w} + \mathbf{T} = \|\mathbf{u}\|\mathbf{i} - \mathbf{j} + \|\mathbf{T}\|(-\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j})$

$\|\mathbf{u}\| = \sin\theta\,\|\mathbf{T}\|$

$1 = \cos\theta\,\|\mathbf{T}\|$

If $\theta = 30°$, $\|\mathbf{u}\| = (1/2)\|\mathbf{T}\|$ and $1 = (\sqrt{3}/2)\|\mathbf{T}\|$

$$\Rightarrow \|\mathbf{T}\| = \frac{2}{\sqrt{3}} \approx 1.1547 \text{ lb} \quad\text{and}\quad \|\mathbf{u}\| = \frac{1}{2}\left(\frac{2}{\sqrt{3}}\right) \approx 0.5774 \text{ lb}$$

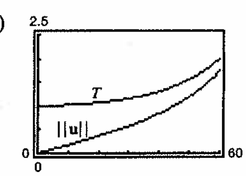
(b) From part (a), $\|\mathbf{u}\| = \tan\theta$ and $\|\mathbf{T}\| = \sec\theta$.

Domain: $0 \le \theta \le 90°$

(c)

θ	0°	10°	20°	30°	40°	50°	60°
T	1	1.0154	1.0642	1.1547	1.3054	1.5557	2
$\|\mathbf{u}\|$	0	0.1763	0.3640	0.5774	0.8391	1.1918	1.7321

(d)



(e) Both are increasing functions.

(f) $\lim_{\theta\to\pi/2^-} T = \infty$ and $\lim_{\theta\to\pi/2^-} \|\mathbf{u}\| = \infty$.

14. (a) The tension T is the same in each tow line.

$$6000\mathbf{i} = T(\cos 20° + \cos(-20))\mathbf{i} + T(\sin 20° + \sin(-20°))\mathbf{j}$$
$$= 2T\cos 20°\mathbf{i}$$
$$\Rightarrow T = \frac{6000}{2\cos 20°} \approx 3192.5 \text{ lbs}$$

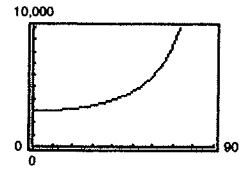
(b) As in part (a), $6000\mathbf{i} = 2T\cos\theta$

$$\Rightarrow T = \frac{3000}{\cos\theta}$$

Domain: $0 < \theta < 90°$

(c)

θ	10°	20°	30°	40°	50°	60°
T	3046.3	3192.5	3464.1	3916.2	4667.2	6000.0

(d) 

(e) As θ increases, there is less force applied in the direction of motion.

15. Let $\theta = \alpha - \beta$, the angle between $\mathbf{u}$ and $\mathbf{v}$. Then

$$\sin(\alpha - \beta) = \frac{\|\mathbf{u} \times \mathbf{v}\|}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{\|\mathbf{v} \times \mathbf{u}\|}{\|\mathbf{u}\|\,\|\mathbf{v}\|}.$$

For $\mathbf{u} = \langle \cos\alpha, \sin\alpha, 0\rangle$ and $\mathbf{v} = \langle \cos\beta, \sin\beta, 0\rangle$, $\|\mathbf{u}\| = \|\mathbf{v}\| = 1$ and

$$\mathbf{v} \times \mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \cos\beta & \sin\beta & 0 \\ \cos\alpha & \sin\alpha & 0 \end{vmatrix} = (\sin\alpha\cos\beta - \cos\alpha\sin\beta)\mathbf{k}.$$

Thus, $\sin(\alpha - \beta) = \|\mathbf{v} \times \mathbf{u}\| = \sin\alpha\cos\beta - \cos\alpha\sin\beta$.

16. (a) Los Angeles: $(4000, -118.24°, 55.95°)$

Rio de Janeiro: $(4000, -43.23°, 112.90°)$

(b) Los Angeles: $x = 4000\sin(55.95°)\cos(-118.24°)$

$y = 4000\sin(55.95°)\sin(-118.24°)$

$z = 4000\cos(55.95°)$

$(x, y, z) \approx (-1568.2, -2919.7, 2239.7)$

Rio de Janeiro: $x = 4000\sin(112.90°)\cos(-43.23°)$

$y = 4000\sin(112.90°)\sin(-43.23°)$

$z = 4000\cos(112.90°)$

$(x, y, z) \approx (2684.7, -2523.8, -1556.5)$

(c) $$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{(-1568.2)(2684.7) + (-2919.7)(-2523.8) + (2239.7)(-1556.5)}{(4000)(4000)} \approx -0.02047$$

$\theta \approx 91.17°$ or 1.59 radians

(d) $s = r\theta = 4000(1.59) \approx 6360$ miles

(e) For Boston and Honolulu:

a. Boston: $(4000, -71.06°, 47.64°)$

Honolulu: $(4000, -157.86°, 68.69°)$

b. Boston: $x = 4000\sin 47.64°\cos(-71.06°)$

$y = 4000\sin 47.64°\sin(-71.06°)$

$z = 4000\cos 47.64°$

$(959.4, -2795.7, 2695.1)$

Honolulu: $x = (4000\sin 68.69°\cos(-157.86°)$

$y = 4000\sin 68.69°\sin(-157.86°)$

$z = 4000\cos 68.69°$

$(-3451.7, -1404.4, 1453.7)$

c. $$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{(959.4)(-3451.7) + (-2795.7)(-1404.4) + (2695.1)(1453.7)}{(4000)(4000)} \approx 0.28329$$

$\theta \approx 73.54°$ or 1.28 radians

d. $s = r\theta = 4000(1.28) \approx 5120$ miles

17. From Theorem 11.13 and Theorem 11.7 (6) we have

$$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|}$$

$$= \frac{|\mathbf{w} \cdot (\mathbf{u} \times \mathbf{v})|}{\|\mathbf{u} \times \mathbf{v}\|} = \frac{|(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}|}{\|\mathbf{u} \times \mathbf{v}\|} = \frac{|\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})|}{\|\mathbf{u} \times \mathbf{v}\|}.$$

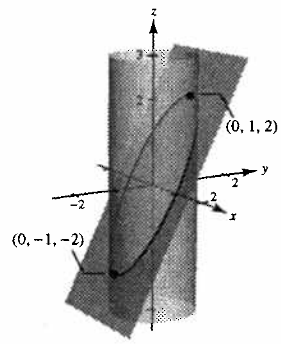
18. Assume one of a, b, c, is not zero, say a. Choose a point in the first plane such as $(-d_1/a, 0, 0)$. The distance between this point and the second plane is

$$D = \frac{|a(-d_1/a) + b(0) + c(0) + d_2|}{\sqrt{a^2 + b^2 + c^2}}$$

$$= \frac{|-d_1 + d_2|}{\sqrt{a^2 + b^2 + c^2}} = \frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}.$$

19. $x^2 + y^2 = 1$ cylinder

$z = 2y$ plane



Introduce a coordinate system in the plane $z = 2y$.

The new u-axis is the original x-axis.

The new v-axis is the line $z = 2y$, $x = 0$.

Then the intersection of the cylinder and plane satisfies the equation of an ellipse:

$$x^2 + y^2 = 1$$

$$x^2 + \left(\frac{z}{2}\right)^2 = 1$$

$$x^2 + \frac{z^2}{4} = 1 \quad \text{ellipse}$$

20. Essay.

CHAPTER 12
Vector-Valued Functions